



**High School  
Honors Physics  
Packet 2  
Eckstrom**

**4TH QUARTER  
CURRICULUM PACKET**

**Hayward Community  
School District  
715-634-2619**

**#HurricaneStrong**



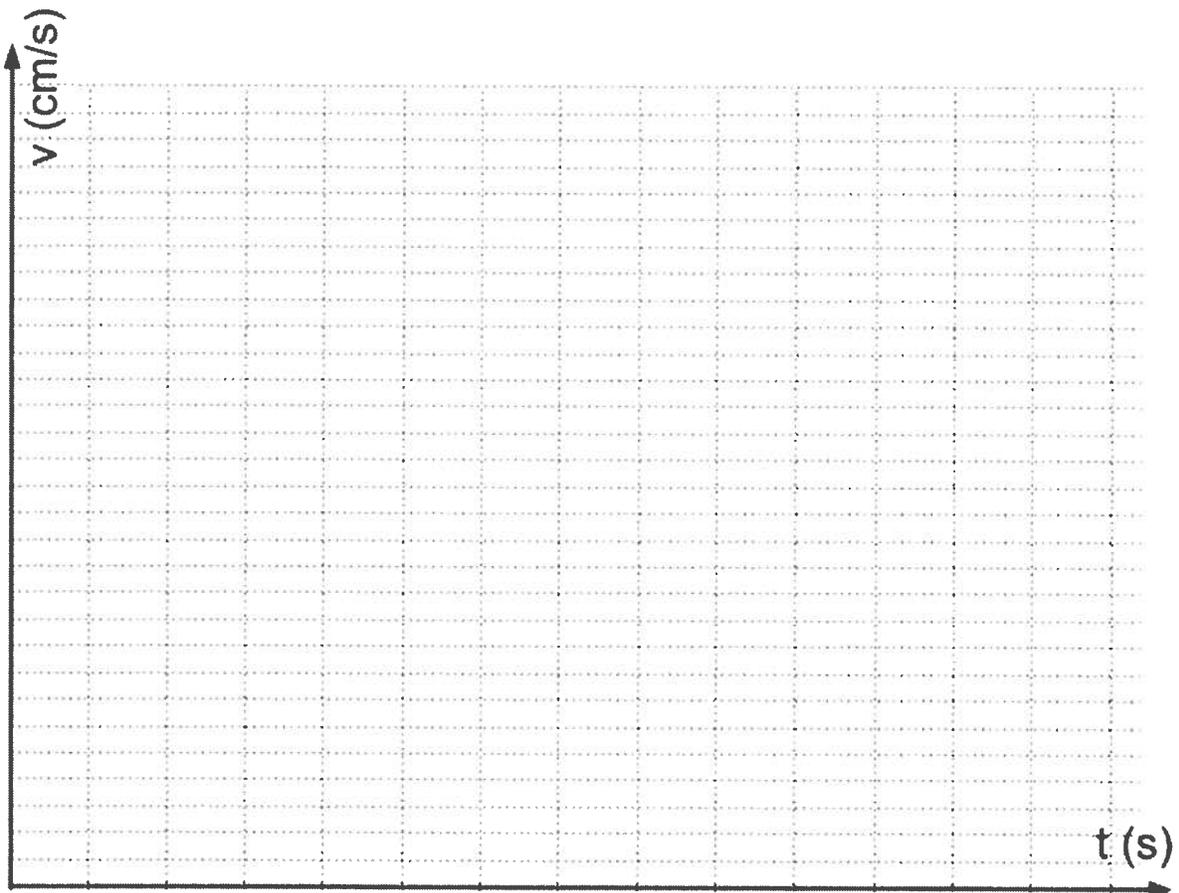
- What does the slope of a position vs. time graph (like the one you just made) mean? Hopefully you remember this from our last unit. If not, look it up.
- You probably noticed that the position vs. time graph you just made does not have a constant slope. How is the slope changing as time goes on?
- What do your last two answers tell you about the motion of the wheel? (Don't describe the graph, think about the actual motion of the actual wheel.)
- On your position vs. time graph, take a straight edge and draw a line that connects the point at  $t = 0$  seconds to the  $t = 6.0$  s.
  - Calculate the slope of this line.
  - What does this slope tell you about the actual motion of the actual wheel?
- Repeat this process of drawing lines and calculating slopes for the following time intervals:
  - $t = 0 \rightarrow t = 2.0$  s
  - $t = 2.0$  s  $\rightarrow t = 4.0$  s
  - $t = 4.0$  s  $\rightarrow t = 6.0$  s
- What do the last 3 slopes tell you about how the motion of the wheel is changing?
- On your graph, draw your best guess at a tangent line to the graph at  $t = 3.0$  s. If you are unsure what a tangent line is, look it up.
  - What is the slope of the tangent line you drew?
  - How does this slope compare to the slope of the line from  $t = 0$  s  $\rightarrow t = 6.0$  s?
- Thinking about your answers to all the questions above, fill in the blank in the following statement:

*If an objects' velocity is changing during a time interval that is  $T$  seconds long where the average velocity is  $V$ , the actual velocity at time  $= \frac{1}{2} T$  is \_\_\_\_\_.*

This next table re-organizes the position and time data a bit in the first two columns.

- The last 4 columns are places where you can put some calculated quantities for the **intervals** between seconds:
  - $\Delta t$ : the length of time between the times in the two rows
  - $\Delta x$ : the change in position between the two rows
  - $t_{mid}$ : exactly halfway between the times in the two rows. Remember, this is the only time the wheel is actually going the average speed during the interval
  - $v_{avg}$ : the average speed over the interval ( $v = \frac{\Delta x}{\Delta t}$ )

t (s)	x (cm)	$\Delta t$ (s)	$\Delta x$ (cm)	$t_{mid}$ (s)	$v_{avg}$ (cm/s)
0.0	0.0				
1.0	5.0				
2.0	20.0				
3.0	45.0				
4.0	80.0				
5.0	125.0				
6.0	180.0				

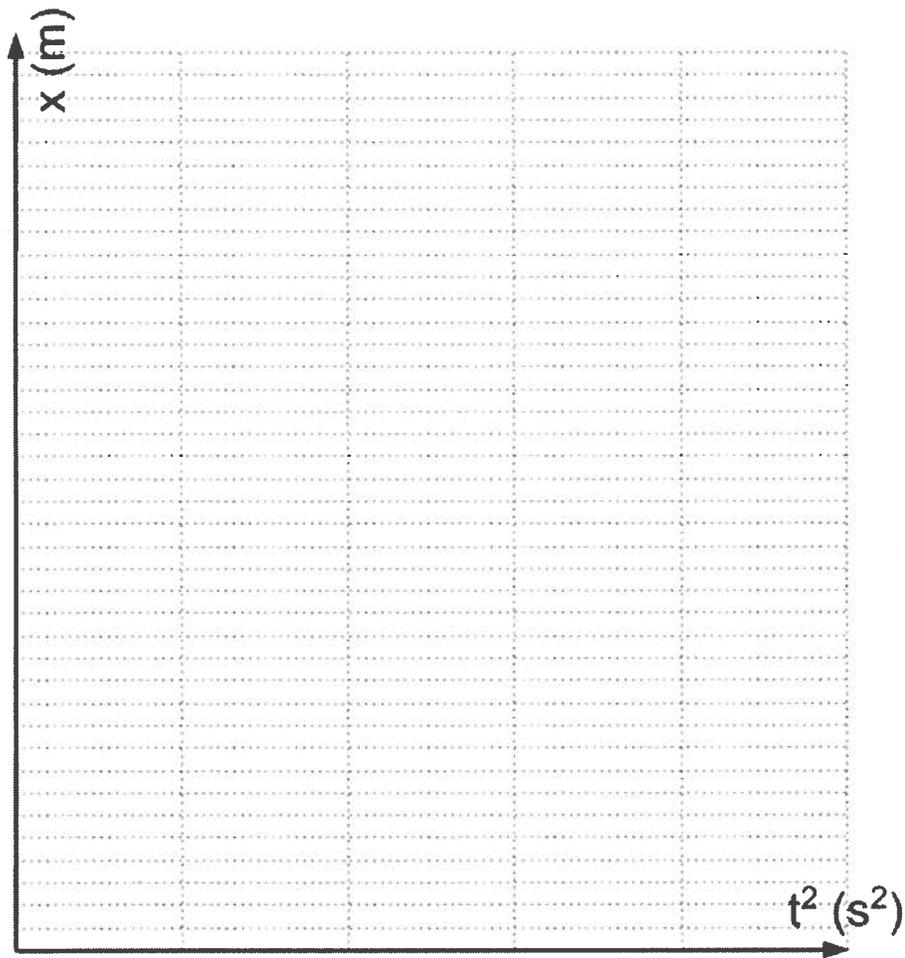


- Let's find a way to describe this graph:
  - Find the slope of the line. Pay attention to units.
  - Write the mathematical equation for this line, including the units. Use  $v$  and  $t$ , not  $x$  and  $y$ !
  - What is the meaning of the slope of this line?
  
- What is the meaning of the area under a velocity vs. time graph? Hopefully you remember this from our last unit. If not, look it up.
  
- What is the area under this graph from  $t = 0$  to  $t = 5.0$  seconds? (If you don't remember how to find the area of this shape, look it up or figure it out.)
  
- Does the area under the graph agree with the data from the position vs. time graph you first made?

- Based on the shape of your position vs. time graph, it should be clear to an experienced model-builder like you that further analysis is needed to find a linear relationship. Let's do that by squaring the time data in the table below.

time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
time <sup>2</sup> (s <sup>2</sup> )							
x (cm)	0.0	5.0	20.0	45.0	80.0	125.0	180.0

- Plot a graph of position vs. time<sup>2</sup>.



- Let's find a way to describe this graph:
  - Find the slope of the line. What are the units of this slope?
  - What is the equation of this line, including units?

- Compare the slope of your  $x$  vs.  $t^2$  line to the slope of your  $v$  vs.  $t$  line.
  - What is the same about the two slopes? (Hint: It's not the numbers.)
  - How do the numerical values of the two slopes compare?
  - Think about what you've learned in this idea-builder by filling in the blanks in this summary paragraph:

The slope of a graph of position vs. time is the \_\_\_\_\_ of the object.

For this wheel on the ramp, the slope \_\_\_\_\_ as time increases, so

this means the wheel's speed is \_\_\_\_\_ as time goes on. The rate of change of velocity as time goes on is called the **acceleration**. Since the graph of velocity vs.

time is linear this means the acceleration is \_\_\_\_\_. On the graph of velocity

vs. time, the acceleration is the \_\_\_\_\_ of the line. The slope of the graph

of position vs.  $t^2$  is \_\_\_\_\_ of the size of the acceleration.

- Select any equations below that are valid mathematical models for an object that has a constantly changing velocity, like this wheel on the ramp. Provide a reason for accepting or rejecting each choice. In this table:

$a$  stands for acceleration,  $x$  stands for position,  $v$  stands for velocity,  $t$  stands for time

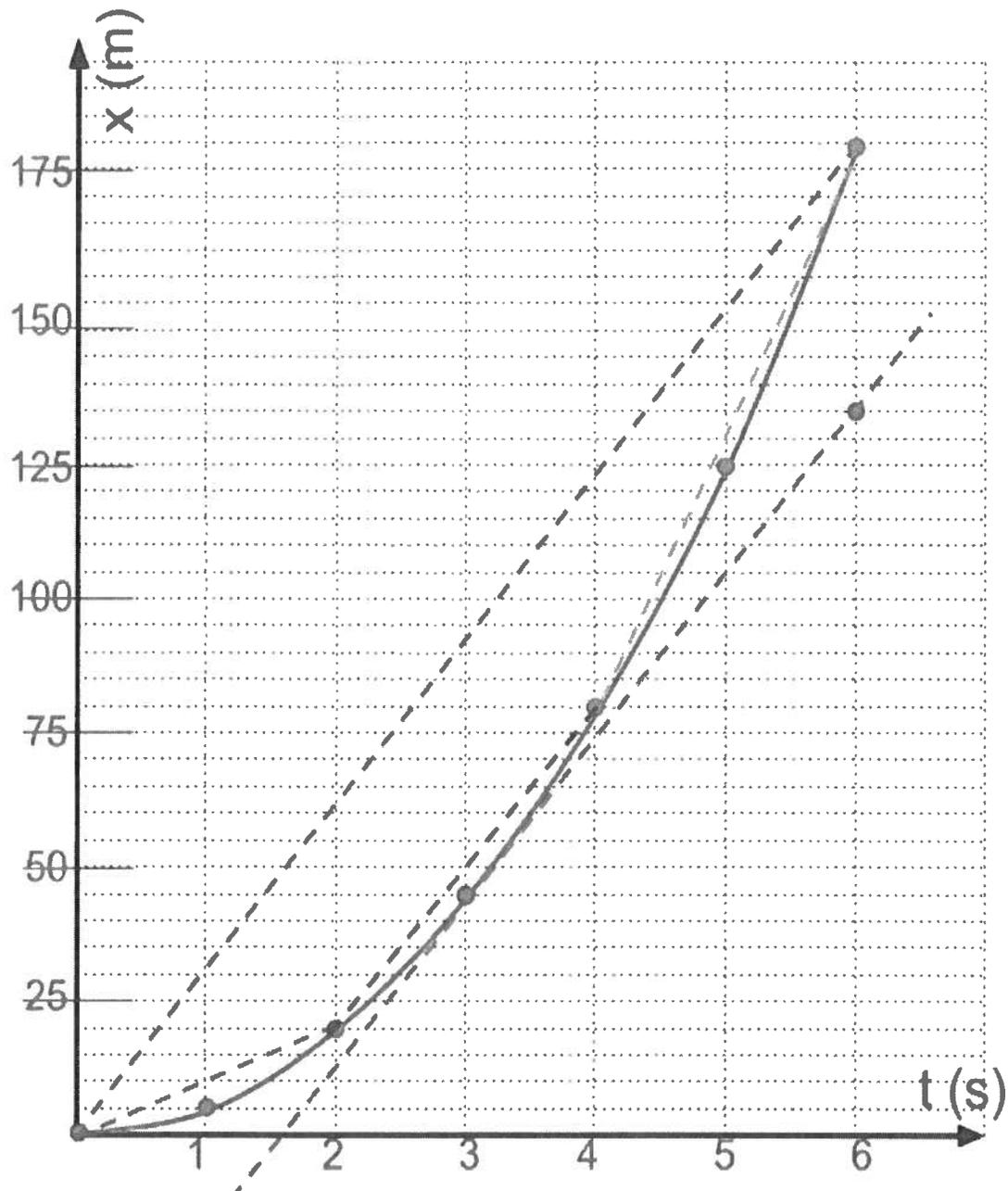
Model	Yes or No	Reason
$x = v \cdot t$		
$v = a \cdot t$		
$x = a \cdot t^2$		
$x = \frac{1}{2} a \cdot t^2$		

## CAPM Idea Builder #1 Solutions

The data in this table are for a wheel rolling from rest down an ramp.

time (s)	0	1.0	2.0	3.0	4.0	5.0	6.0
x (cm)	0.0	5.0	20.0	45.0	80.0	125.0	180.0

- Plot the position vs. time graph for this motion:



- What does the slope of a position vs. time graph (like the one you just made) mean? Hopefully you remember this from our last unit. If not, look it up. The slope of a position vs. time graph is the velocity of the object.
- You probably noticed that the position vs. time graph you just made does not have a constant slope. How is the slope changing as time goes on? The slope appears to be getting steeper.
- What do your last two answers tell you about the motion of the wheel? (Don't describe the graph, think about the actual motion of the actual wheel.) Since the steepness of the line represents the speed of the object and the steepness is increasing, the object is speeding up.
- On your position vs. time graph, take a straight edge and draw a line that connects the point at  $t = 0$  seconds to the  $t = 6.0$  s.

- Calculate the slope of this line. This is shown as a red dashed line.

$$v = \frac{\Delta x}{\Delta t} = \frac{180 \text{ cm}}{6 \text{ s}} = 30 \frac{\text{cm}}{\text{s}}$$

- What does this slope tell you about the actual motion of the actual wheel? Even though its velocity is changing, on average, the wheel is going 30 m/s.
- Repeat this process of drawing lines and calculating slopes for the following time intervals:

- $t = 0 \rightarrow t = 2.0$  s This is shown as a green dashed line:  $v = \frac{20 \text{ cm}}{2 \text{ s}} = 10 \frac{\text{cm}}{\text{s}}$

- $t = 2.0$  s  $\rightarrow t = 4.0$  s This is shown as a blue dashed line:  $v = \frac{60 \text{ cm}}{2 \text{ s}} = 30 \frac{\text{cm}}{\text{s}}$

- $t = 4.0$  s  $\rightarrow t = 6.0$  s This is an orange dashed line:  $v = \frac{100 \text{ cm}}{2 \text{ s}} = 50 \frac{\text{cm}}{\text{s}}$

- What do the last 3 slopes tell you about how the motion of the wheel is changing? The wheel appears to be increasing its average speed by 20 cm/s every 2 seconds.
- On your graph, draw your best guess at a tangent line to the graph at  $t = 3.0$  s. If you are unsure what a tangent line is, look it up. This is shown as a purple dashed line.

- What is the slope of the tangent line you drew? Using the two purple dots as my

$$\text{reference: } v = \frac{(135 \text{ cm} - 45 \text{ cm})}{(6 \text{ s} - 3 \text{ s})} = \frac{90 \text{ cm}}{3 \text{ s}} = 30 \frac{\text{cm}}{\text{s}}$$

- How does this slope compare to the slope of the line from  $t = 0$  s  $\rightarrow t = 6.0$  s? It's the same
- Thinking about your answers to all the questions above, fill in the blank in the following statement:

*If an object's velocity is changing during a time interval that is  $T$  seconds long where the average velocity is  $V$ , the actual velocity at time  $= \frac{1}{2} T$  is  $V$ , because it is half-way through the change in velocity the velocity is the average velocity.*

This next table re-organizes the position and time data a bit in the first two columns.

- The last 4 columns are places where you can put some calculated quantities for the ***intervals*** between seconds: I did the first one here
  - $\Delta t$ : the length of time between the times in the two rows  
 $1.0\text{ s} - 0.0\text{ s} = 1.0\text{ s}$
  - $\Delta x$ : the change in position between the two rows

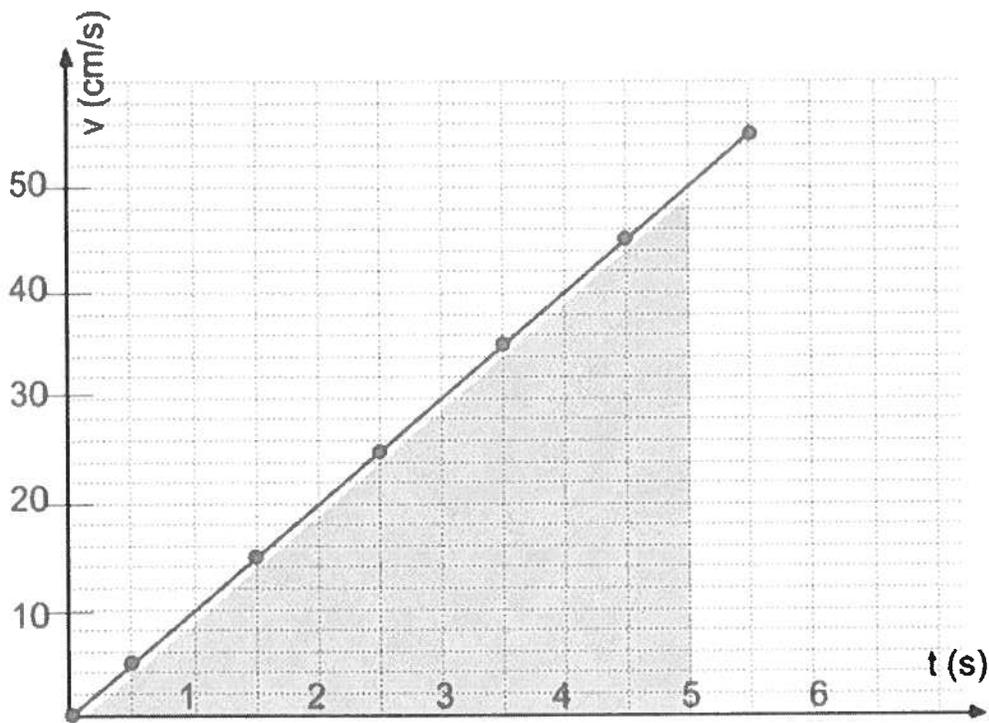
t (s)	x (cm)	$\Delta t$ (s)	$\Delta x$ (cm)	$t_{\text{mid}}$ (s)	$v_{\text{avg}}$ (cm/s)
0.0	0.0	↔ 1.0	5.0	0.5	5.0
1.0	5.0	↔ 1.0	15.0	1.5	15
2.0	20.0	↔ 1.0	25.0	2.5	25
3.0	45.0	↔ 1.0	35.0	3.5	35
4.0	80.0	↔ 1.0	45.0	4.5	45
5.0	125.0	↔ 1.0	55.0	5.5	55
6.0	180.0				

$$5.0\text{ cm} - 0.0\text{ cm} = 5.0\text{ cm}$$

- $t_{\text{mid}}$ : exactly halfway between the times in the two rows. Remember, this is the only time the wheel is actually going the average speed during the interval

$$0\text{ s} + \frac{1.0\text{ s}}{2} = 0.5\text{ s}$$

- $v_{\text{avg}}$ : the average speed over the interval ( $v = \frac{\Delta x}{\Delta t}$ )  $\frac{5.0\text{ cm}}{1.0\text{ s}} = 5.0\frac{\text{cm}}{\text{s}}$



- Let's find a way to describe this graph:

- Find the slope of the line. Pay attention to units.  $\frac{55 \frac{cm}{s}}{5.5 s} = 10 \frac{cm}{s} = 10 \frac{cm}{s^2}$
- Write the mathematical equation for this line, including the units. Use v and t, not x and y!  $v = 10 \frac{cm}{s} \cdot t$
- What is the meaning of the slope of this line? The slope of this line tells how fast the velocity is changing over time.

- What is the meaning of the area under a velocity vs. time graph? Hopefully you remember this from our last unit. If not, look it up. It's the displacement.
- What is the area under this graph from  $t = 0$  to  $t = 5.0$  seconds? (If you don't remember how to find the area of this shape, look it up or figure it out.) This area is shaded in pink. A triangle is just half of a rectangle, so its area is half the width times the height.

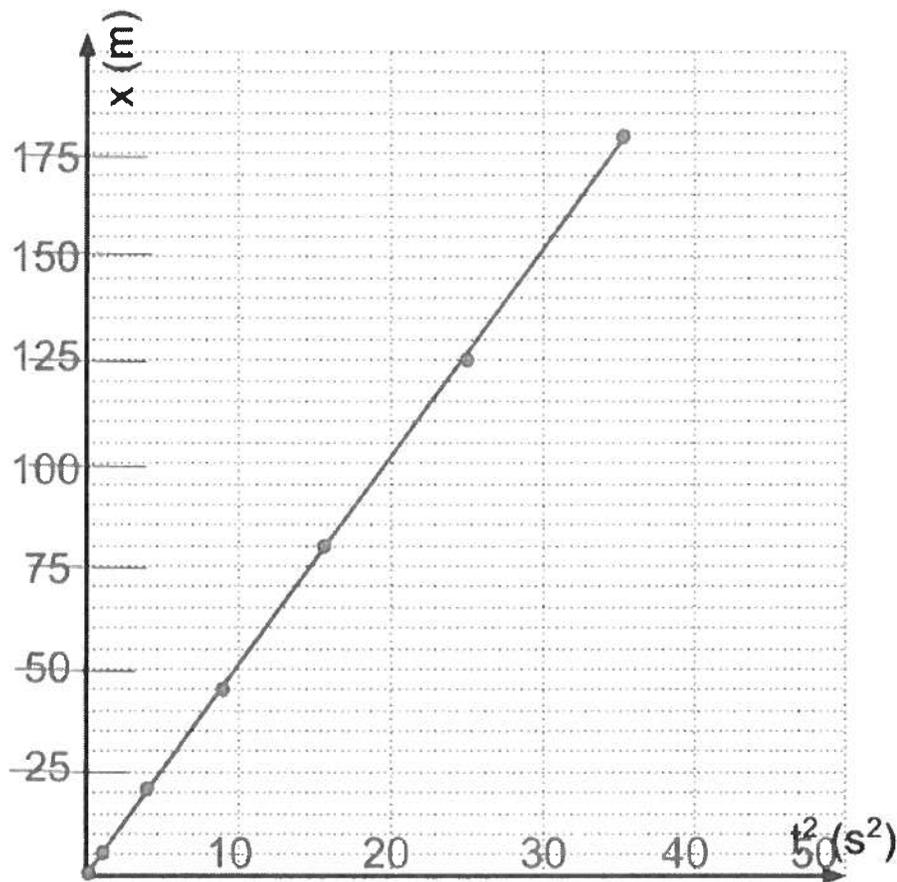
$$\frac{\left(50 \frac{cm}{s}\right) (5 s)}{2} = 125 cm$$

- Does the area under the graph agree with the data from the position vs. time graph you first made? Yes. On the x vs. t graph, the position is 125 cm when the time is 5.0 s.

- Based on the shape of your position vs. time graph, it should be clear to an experienced model-builder like you that further analysis is needed to find a linear relationship. Let's do that by squaring the time data in the table below.

time (s)	0.0	1.0	2.0	3.0	4.0	5.0	6.0
time <sup>2</sup> (s <sup>2</sup> )	0.0	1.0	4.0	9.0	16	25	36
x (cm)	0.0	5.0	20.0	45.0	80.0	125.0	180.0

- Plot a graph of position vs. time<sup>2</sup>.



- Let's find a way to describe this graph:

- Find the slope of the line. What are the units of this slope?  $\frac{180 \text{ cm}}{36 \text{ s}^2} = 5.0 \frac{\text{cm}}{\text{s}^2}$

- What is the equation of this line, including units?  $x = \left(5.0 \frac{\text{cm}}{\text{s}^2}\right) t^2$

- Compare the slope of your  $x$  vs.  $t^2$  line to the slope of your  $v$  vs.  $t$  line.
  - What is the same about the two slopes? (Hint: It's not the numbers.) The units of slope for both can be expressed as  $\frac{cm}{s^2}$ .
  - How do the numerical values of the two slopes compare? The slope of the  $x$  vs.  $t^2$  line is half the slope of the  $v$  vs.  $t$  graph.
  - Think about what you've learned in this idea-builder by filling in the blanks in this summary paragraph:

The slope of a graph of position vs. time is the \_\_\_\_\_velocity\_\_\_\_\_ of the object.

For this wheel on the ramp, the slope \_\_\_\_\_gets steeper\_\_\_\_\_ as time increases, so

this means the wheel's speed is \_\_\_\_\_increasing\_\_\_\_\_ as time goes on. The rate of change of velocity as time goes on is called the **acceleration**. Since the graph of velocity vs.

time is linear this means the acceleration is \_\_\_\_\_constant\_\_\_\_\_. On the graph of velocity

vs. time, the acceleration is the \_\_\_\_\_slope\_\_\_\_\_ of the line. The slope of the graph

of position vs. time<sup>2</sup> is \_\_\_half\_\_\_ of the size of the acceleration.

- Select any equations below that are valid mathematical models for an object that has a constantly changing velocity, like this wheel on the ramp. Provide a reason for accepting or rejecting each choice. In this table:

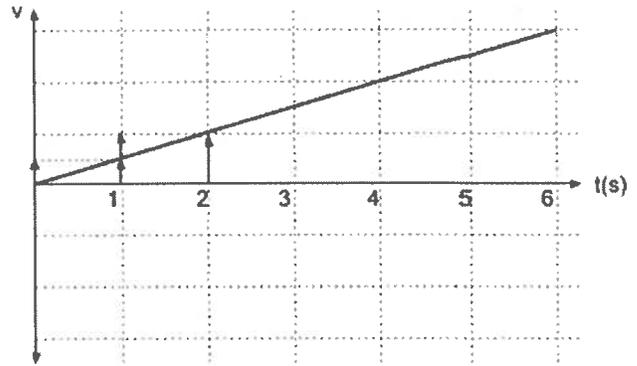
$a$  stands for acceleration,  $x$  stands for position,  $v$  stands for velocity,  $t$  stands for time

Model	Yes or No	Reason
$x = v \cdot t$	No	This is for CVPM and won't work when $v$ isn't constant.
$v = a \cdot t$	Yes	This is just the general equation for the $v$ vs. $t$ line
$x = a \cdot t^2$	No	The slope of the graph of $x$ vs. $t^2$ was half of $a$ .
$x = \frac{1}{2}a \cdot t^2$	Yes	This is the equation of the graph of $x$ vs. $t^2$ .



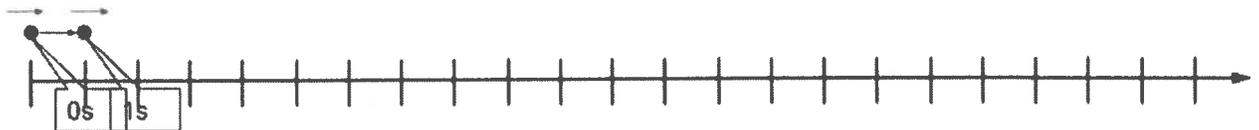
## CAPM Idea Builder #2

- Look at the velocity vs. time graph shown.



- Describe the motion of this object.
- On the graph, draw a blue arrow from the time axis ( $v = 0$  line) to the velocity line at each second from  $t = 0$  to  $t = 6$  s. The first couple have been done as an example. (Ignore the red arrows for now.)
  - What do the length and direction of each blue arrow represent?
  - Why is there no blue arrow at  $t = 0$ ?
  - What do you notice about how the lengths of the blue arrows are changing? What does this mean about the motion of the object?

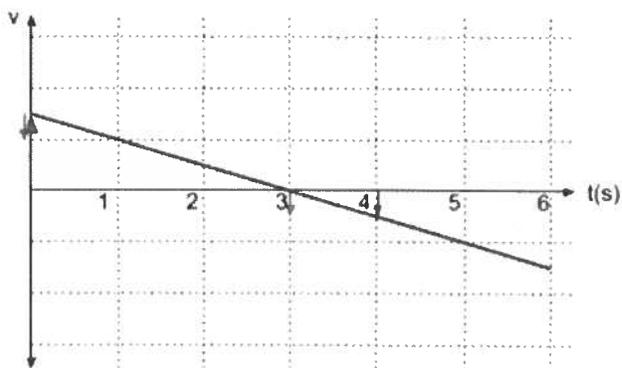
- Draw a motion map representing the motion of this object. Use the lengths of the blue arrows you drew to find the position changes for the dots from one second to the next. For example, the average velocity for the first second is  $\frac{1}{2}$  unit on the  $v$  vs.  $t$  graph, so the length of the vector that connects the 0 s and 1 s dot is  $\frac{1}{2}$  unit long (This one is done for you. Ignore the red arrows for now.)



- Velocity is changing over time. At  $t = 0$  and  $t = 1$  s, red arrows have been drawn.
  - What change does the length of these arrows represent? (The dotted line on the first one is a hint.) Draw the rest of the arrows.
  - What do the size and direction of these arrows represent?
  - What physics vector quantity do these arrows represent?
  - At each second on the motion map, add the red vector that shows how the velocity is going to change in that next second. The first two are in place already.

- Look at the velocity vs. time graph shown.

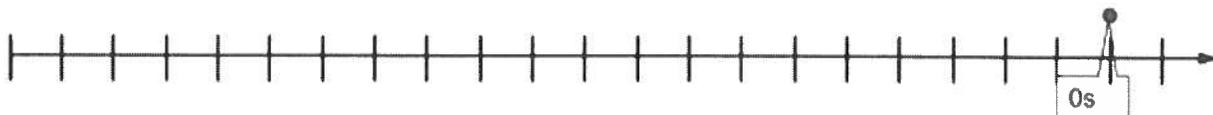
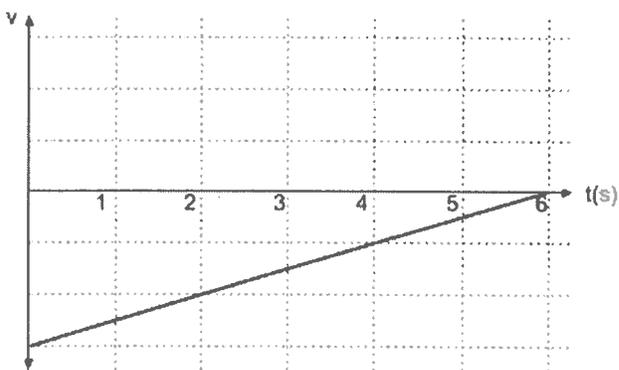
- Describe the motion of this object.
- On the graph, draw the blue velocity vectors. The first one and the one at 4 s have been done for you.
- On the graph, draw the red acceleration vectors. The first one and the one at 3 s have been done for you.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down?

- Look at the velocity vs. time graph shown.

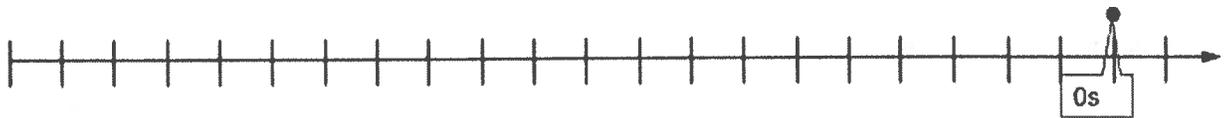
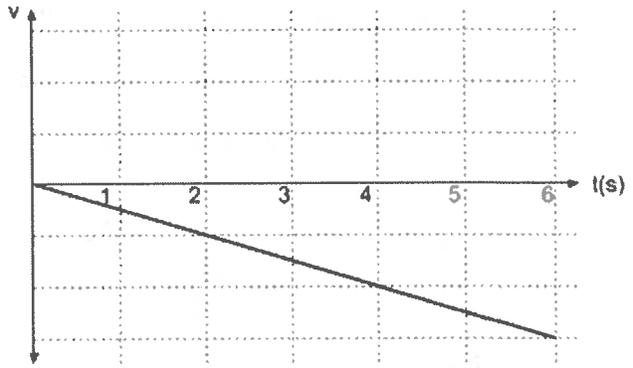
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down?

- Look at the velocity vs. time graph shown.

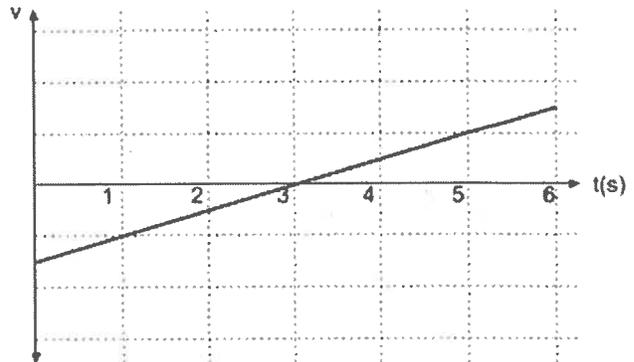
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down?

- Look at the velocity vs. time graph shown.

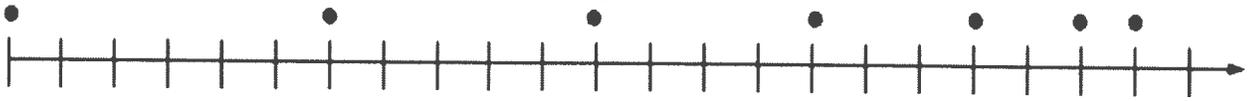
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



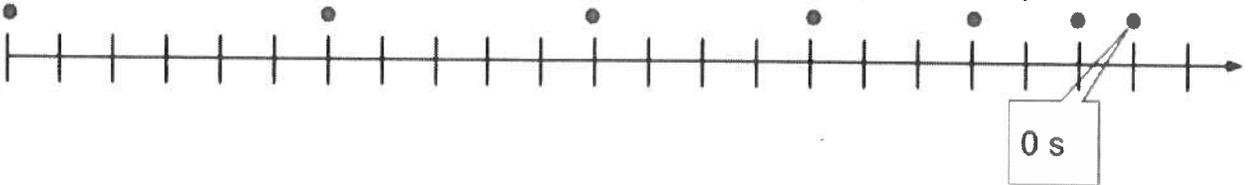
- Is the object speeding up or slowing down?

Let's summarize:

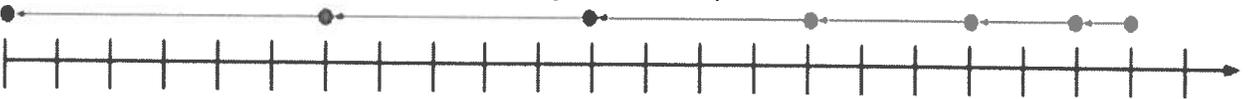
- On a motion map like this one, if you only can see the dots, can you tell if the object is speeding up or slowing down? Explain.



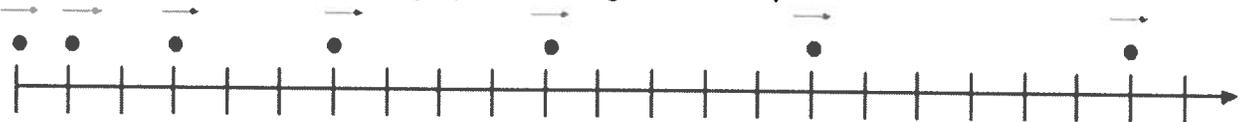
- On a motion map like this one, if you only can see the dots and you know the starting dot, can you tell if the object is speeding up or slowing down? Explain.



- On a motion map like this one, if you can only see the velocity vectors, can you tell if an object is speeding up or slowing down? Explain.



- On a motion map like this one, if you can only see the acceleration vectors, can you tell if an object is speeding up or slowing down? Explain.

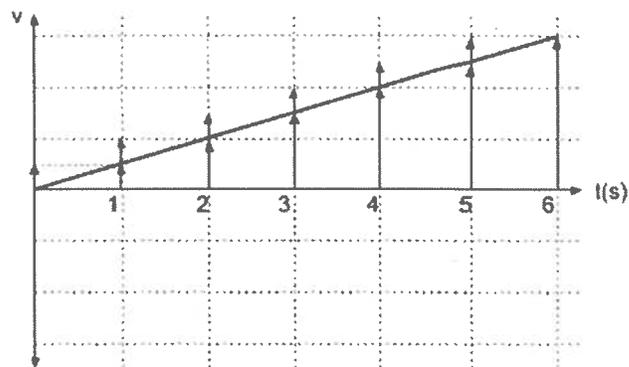


- Fill in the blanks in the statements below with a phrase that makes the statement true:

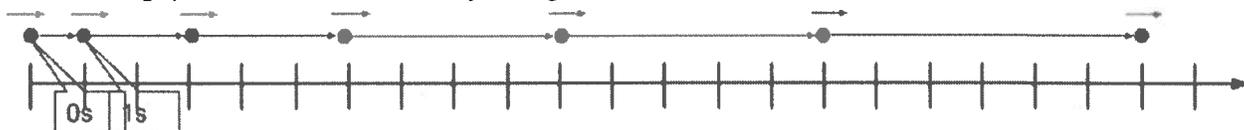
- If the object is speeding up, the directions of the acceleration vectors and the velocity vectors point \_\_\_\_\_.
- If the object is slowing down, the directions of the acceleration vectors and the velocity vectors point \_\_\_\_\_.

## CAPM Idea Builder #2

- Look at the velocity vs. time graph shown.



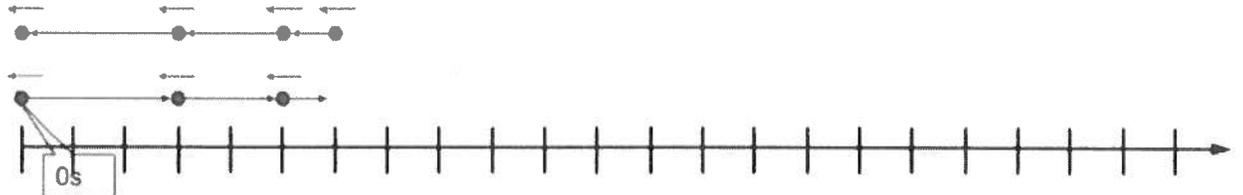
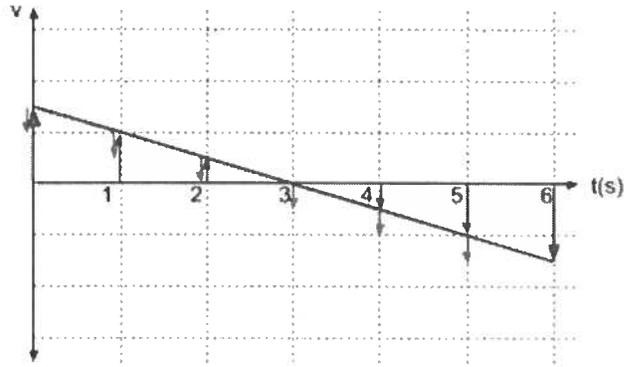
- Describe the motion of this object. It's moving forward at increasing speed.
- On the graph, draw a blue arrow from the time axis ( $v = 0$  line) to the velocity line at each second from  $t = 0$  to  $t = 6$  s. The first couple have been done as an example. (Ignore the red arrows for now.)
  - What do the length and direction of each blue arrow represent? The length represents the speed. The direction represents the direction of motion.
  - Why is there no blue arrow at  $t = 0$ ? The velocity at 0 is 0.
  - What do you notice about how the lengths of the blue arrows are changing? What does this mean about the motion of the object? The arrows get longer by the same amount each time. This means the object's velocity is increasing at a constant rate.
- Draw a motion map representing the motion of this object. Use the lengths of the blue arrows you drew to find the position changes for the dots from one second to the next. For example, the average velocity for the first second is  $\frac{1}{2}$  unit on the  $v$  vs.  $t$  graph, so the length of the vector that connects the 0 s and 1 s dot is  $\frac{1}{2}$  unit long (This one is done for you. Ignore the red arrows for now.)



- Velocity is changing over time. At  $t = 0$  and  $t = 1$  s, red arrows have been drawn.
  - What change does the length of these arrows represent? (The dotted line on the first one is a hint.) Draw the rest of the arrows.
  - What do the size and direction of these arrows represent? The length represents the size of the velocity change. The direction represents the direction of velocity change.
  - What physics vector quantity do these arrows represent? The velocity change each second is the acceleration.
  - At each second on the motion map, add the red vector that shows how the velocity is going to change in that next second. The first two are in place already.

- Look at the velocity vs. time graph shown.

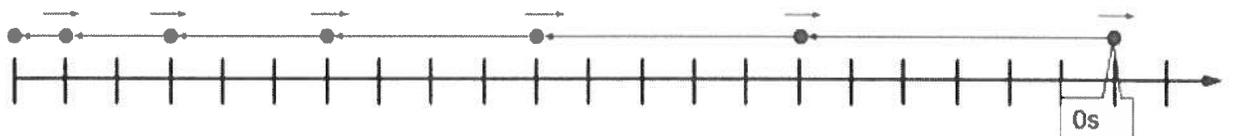
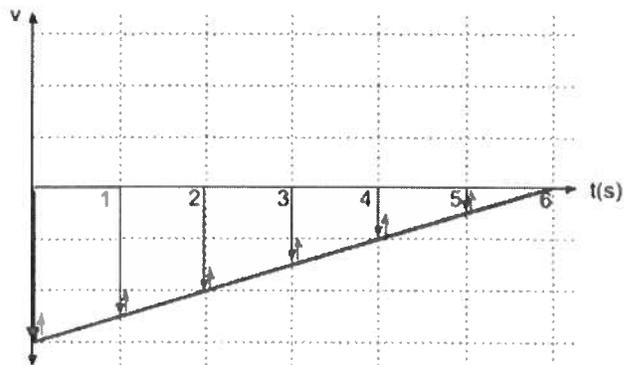
- Describe the motion of this object.
- On the graph, draw the blue velocity vectors. The first one and the one at 4 s have been done for you.
- On the graph, draw the red acceleration vectors. The first one and the one at 3 s have been done for you.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down? It slows down at first, then speeds up after changing directions.

- Look at the velocity vs. time graph shown.

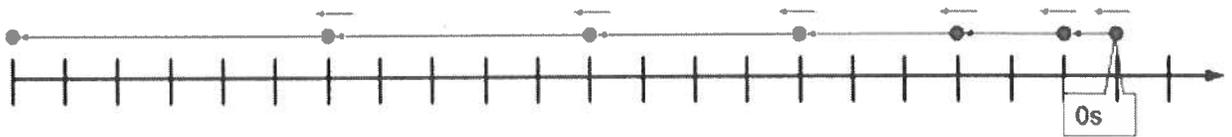
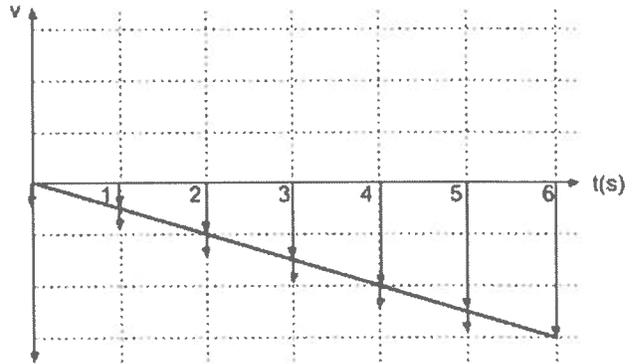
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down? The object is slowing down.

- Look at the velocity vs. time graph shown.

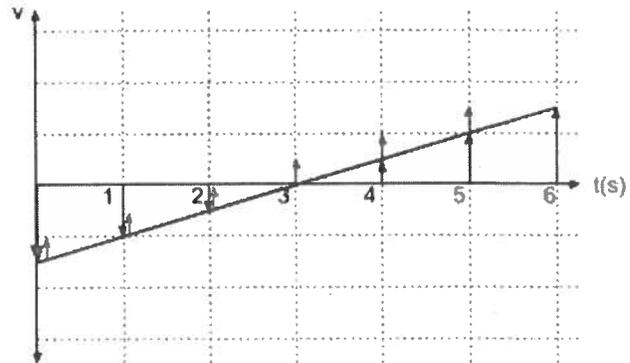
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down? The object is speeding up.

- Look at the velocity vs. time graph shown.

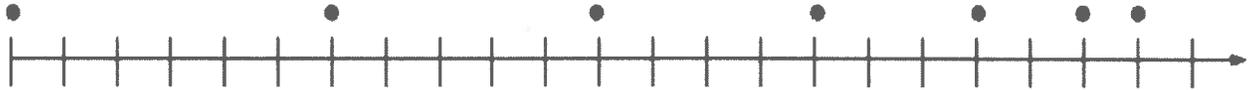
- On the graph, draw the blue velocity vectors.
- On the graph, draw the red acceleration vectors.
- Create a motion map representing the motion of this object. Include both the velocity vectors and the acceleration vectors. The first dot has been placed for you.



- Is the object speeding up or slowing down? The object slows down going backwards. Then it stops and speeds up going forwards.

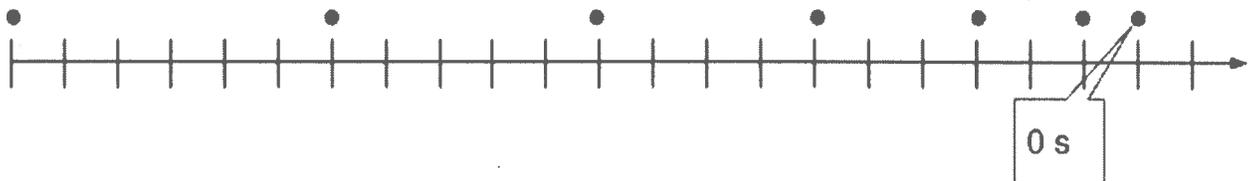
Let's summarize:

- On a motion map like this one, if you only can see the dots, can you tell if the object is speeding up or slowing down? Explain.



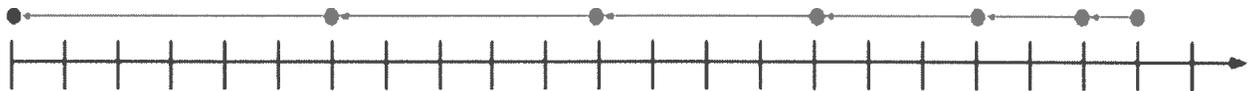
It's impossible to tell if the object is speeding up or slowing down. If it's going forward it's slowing down. But if it's going backwards it is speeding up.

- On a motion map like this one, if you only can see the dots and you know the starting dot, can you tell if the object is speeding up or slowing down? Explain.



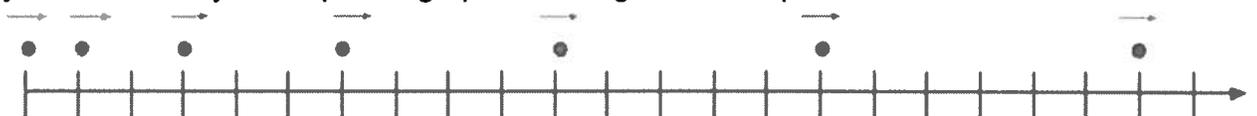
Now that we know where the first second is, we know the object is moving backwards. Since the dots are getting farther apart, this means the object is speeding up.

- On a motion map like this one, if you can only see the velocity vectors, can you tell if an object is speeding up or slowing down? Explain.



The velocity vectors are getting longer, so therefore the object is speeding up.

- On a motion map like this one, if you can only see the acceleration vectors, can you tell if an object is speeding up or slowing down? Explain.



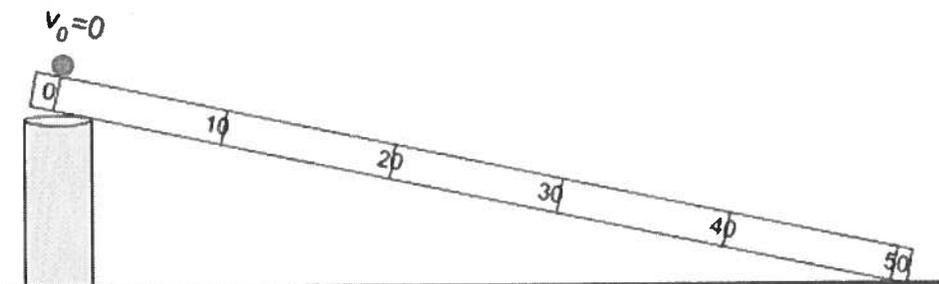
With only the acceleration vectors, it is impossible to tell if this object is speeding up or slowing down. If it is going backwards, it is slowing down. If it is going forwards it is speeding up. This tells us can a positive acceleration can mean speeding up but it can also mean slowing down.

- Fill in the blanks in the statements below with a phrase that makes the statement true:
  - If the object is speeding up, the directions of the acceleration vectors and the velocity vectors point In the same direction.
  - If the object is slowing down, the directions of the acceleration vectors and the velocity vectors point In opposite directions.

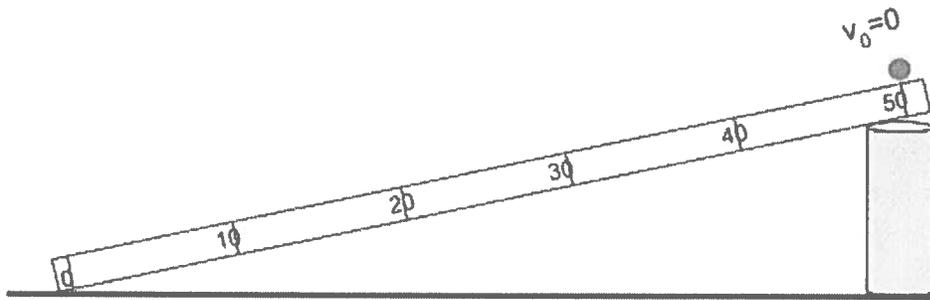
# CAPM Practice and Exploration #1

Name \_\_\_\_\_

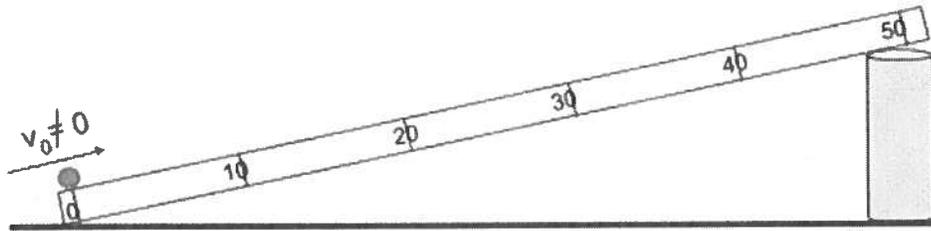
Each physical situation represented below can be modeled a number of different ways. Each box in the table below the sketch asks you for a different representation. In the boxes where mathematical representations are requested, just use symbols for constants like slope or vertical intercept, since we can't calculate many of these.



	Math expression for $x$ vs. $t$ :
	Math expression for $v$ vs. $t$ :
	Math expression for $a$ vs. $t$ :
<p>Motion map:</p>	

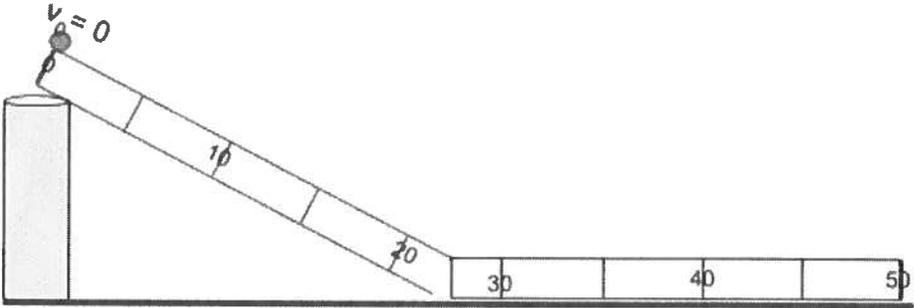


<p>A graph with position <math>x</math> on the vertical axis and time <math>t</math> on the horizontal axis. The axes are shown but no data is plotted.</p>	<p>Math expression for <math>x</math> vs. <math>t</math>:</p>
<p>A graph with velocity <math>v</math> on the vertical axis and time <math>t</math> on the horizontal axis. The axes are shown but no data is plotted.</p>	<p>Math expression for <math>v</math> vs. <math>t</math>:</p>
<p>A graph with acceleration <math>a</math> on the vertical axis and time <math>t</math> on the horizontal axis. The axes are shown but no data is plotted.</p>	<p>Math expression for <math>a</math> vs. <math>t</math>:</p>
<p>Motion map:</p> <p>A horizontal axis for a motion map with tick marks at 0, 10, 20, 30, 40, and 50.</p>	

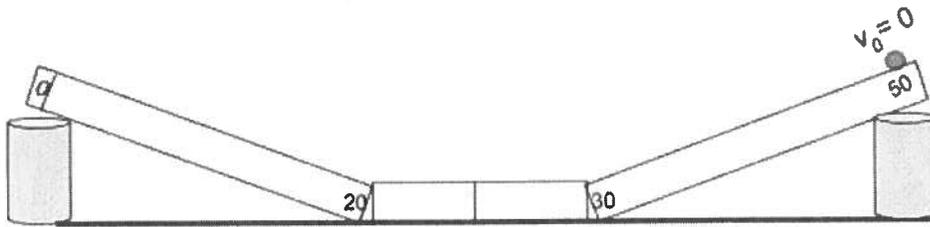


	Math expression for x vs. t:
	Math expression for v vs. t:
	Math expression for a vs. t:
Motion map: 	

In this situation, assume that the ball does not experience any change in velocity while it is on the horizontal part of the ramp.



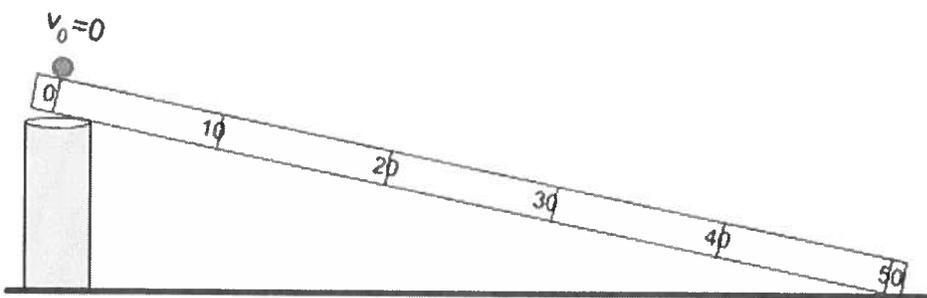
	Math expression for x vs. t:	
	Before 25	After 25
	Math expression for v vs. t:	
	Before 25	After 25
	Math expression for a vs. t:	
	Before 25	After 25
Motion map:		

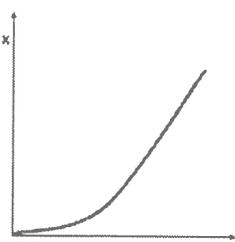
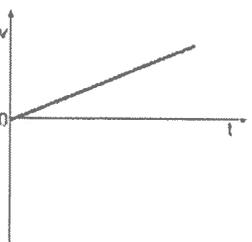


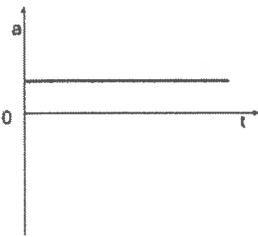
	Math expression for x vs. t:		
	50 → 30	30 → 20	20 → 0
	Math expression for v vs. t:		
	50 → 30	30 → 20	20 → 0
	Math expression for a vs. t:		
	50 → 30	30 → 20	20 → 0
Motion map:			

## CAPM Practice and Exploration #1 Solutions

Each physical situation represented below can be modeled a number of different ways. Each box in the table below the sketch asks you for a different representation. In the boxes where mathematical representations are requested, just use symbols for constants like slope or vertical intercept, since we can't calculate many of these.



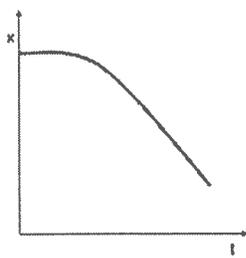
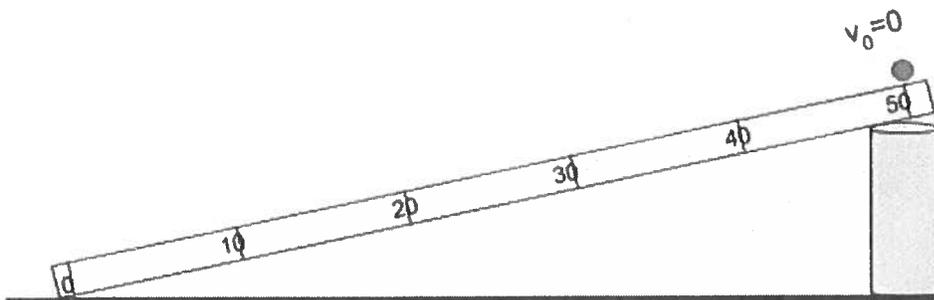
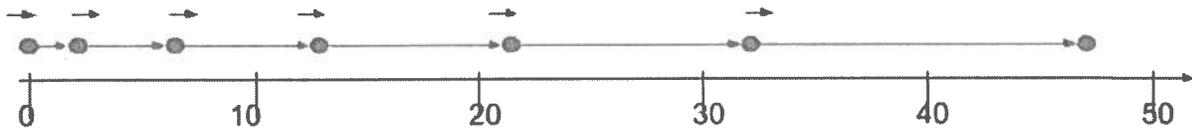
 <p>As the ball rolls down the ramp, influenced by gravity, its velocity will increase steadily. Since the slope represents velocity on a <math>x</math> vs. <math>t</math> graph, this means the slope will be steadily increasing.</p>	<p><b>Math expression for <math>x</math> vs. <math>t</math>:</b> We learned that the shape in the graph has a general equation in which the vertical axis variable depends on the square of the horizontal axis variable, so <math>x = mt^2</math>. But our analysis has shown us that the constant is half the value of the acceleration, so <math>x = \left(\frac{a}{2}\right)t^2</math></p>
 <p>The velocity is always positive, because the slope of <math>x</math> vs. <math>t</math> is always positive, but it is increasing steadily over time.</p>	<p><b>Math expression for <math>v</math> vs. <math>t</math>:</b> This is the equation of a line, where the vertical intercept is 0. We have defined the slope of the <math>v</math> vs. <math>t</math> graph is the acceleration, so <math>v = at</math></p>



Since acceleration is the slope of  $v$  vs.  $t$ , and that graph shows a constant, positive slope, this graph should be a horizontal line above zero.

**Math expression for  $a$  vs.  $t$ :** The slope of this line is 0 and the vertical intercept is some constant value, which I'll just call " $k$ " for lack of an actual number:  $a = k$

**Motion map:** This should show dots that are spaced increasingly far apart and  $v$  vectors that point in the positive direction always, but increase in length. The  $a$  vectors are also positive, because acceleration is positive, and they are the same length because acceleration is constant:

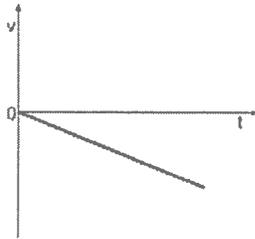


As the ball rolls down the ramp, influenced by gravity, its speed will increase steadily. Since the steepness of slope represents speed on a  $x$  vs.  $t$  graph, this means the steepness will be steadily increasing. Unlike in the first example, the direction is backward,

**Math expression for  $x$  vs.  $t$ :** The general equation for this is explained in the first example. What's different here is that the starting position wasn't zero, so we need to add the starting position to the change:

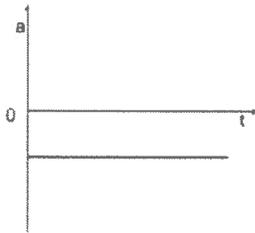
$$x = x_0 + \left(\frac{a}{2}\right)t^2$$

so the slope of the  $x$  vs.  $t$  graph will be negative at all times.



Here, the velocity is always negative, but the speed is increasing, so we show the velocity getting farther away from the  $v=0$  line as time increases.

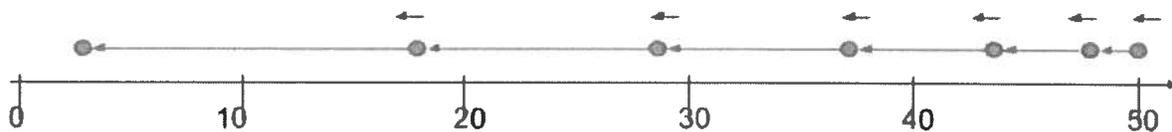
**Math expression for  $v$  vs.  $t$ :** This is the equation of a line, where the vertical intercept is 0. We have defined the slope of the  $v$  vs.  $t$  graph is the acceleration, so  $v = at$

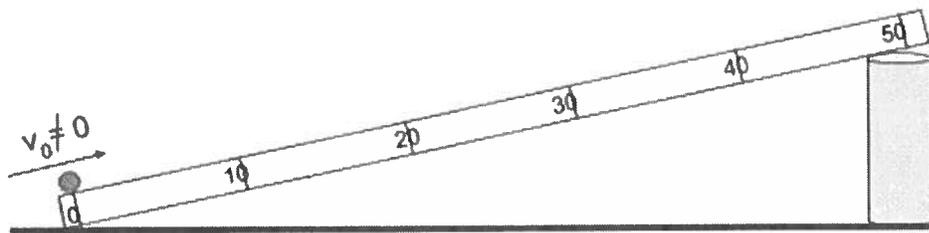


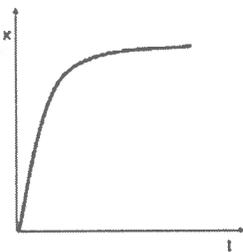
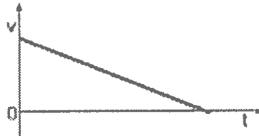
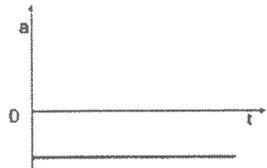
Since acceleration is the slope of  $v$  vs.  $t$ , and that graph shows a constant, negative slope, this graph should be a horizontal line below zero.

**Math expression for  $a$  vs.  $t$ :** The slope of this line is 0 and the vertical intercept is some constant value, which I'll just call " $k$ " for lack of an actual number:  $a = k$

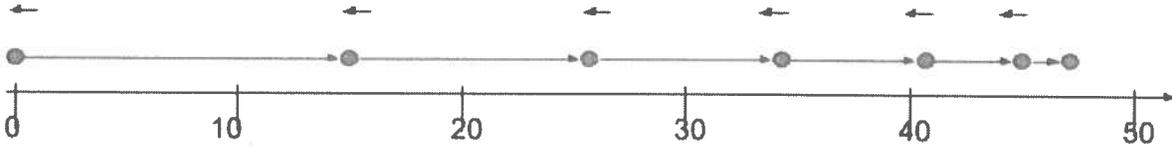
**Motion map:** This should show dots that are space increasingly far apart and  $v$  vectors that point in the negative direction always, but increase in length. The  $a$  vectors are also negative, because acceleration is negative, and they are the same length because acceleration is constant:



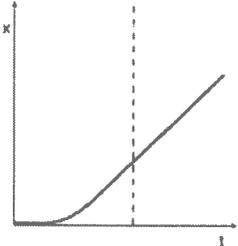
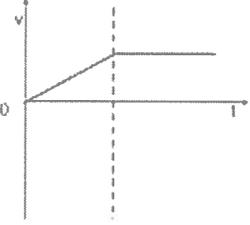


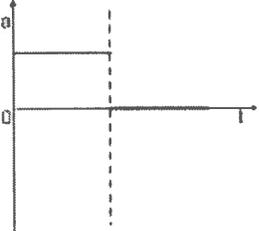
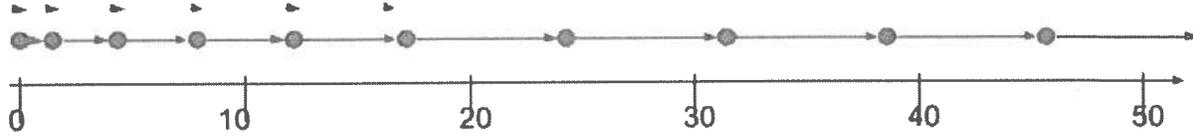
 <p>Unlike in the first two examples, the ball here has a velocity that isn't zero at the start. That means there is a slope (positive in this case) at first. The ball will slow down but still have a positive velocity until it comes to a stop (that's where I'll stop tracking it). This means the slope should get lower and lower until it is zero.</p>	<p><b>Math expression for x vs. t:</b> We have learned that the entire general expression for position vs. time is <math>x = x_0 + v_0 t + \left(\frac{a}{2}\right) t^2</math>. The equations in the two above examples were special cases where, first, both <math>x_0</math> and <math>v_0</math> were zero and, second, <math>x_0</math> was not zero, but <math>v_0</math> was. Now <math>x_0</math> is zero, but <math>v_0</math> is not: <math>x = v_0 t + \left(\frac{a}{2}\right) t^2</math></p>
 <p>The velocity here is positive at time zero, but it steadily decreases to zero, so this graph should reflect that.</p>	<p><b>Math expression for v vs. t:</b> Now the equation of this line includes a non-zero vertical intercept: <math>v = v_0 + at</math></p>
 <p>Since acceleration is the slope of v vs. t, and that graph shows a constant, negative slope, this graph should be a horizontal line below zero.</p>	<p><b>Math expression for a vs. t:</b> The slope of this line is 0 and the vertical intercept is some constant value, which I'll just call "k" for lack of an actual number: <math>a = k</math></p>

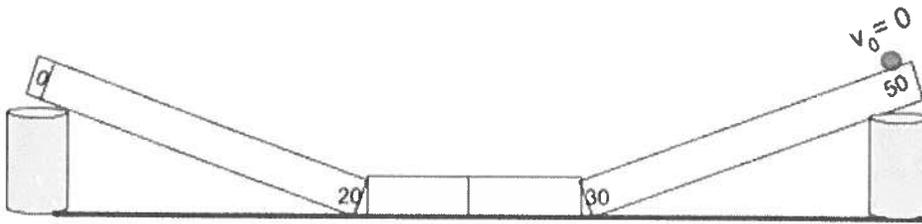
**Motion map:** This should show dots that are space increasingly close together and  $v$  vectors that point in the positive direction always, but decrease in length. The  $a$  vectors are negative, because acceleration is negative, and they are the same length because acceleration is constant:

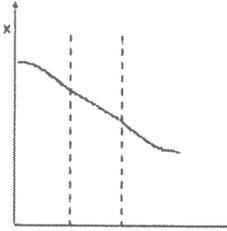


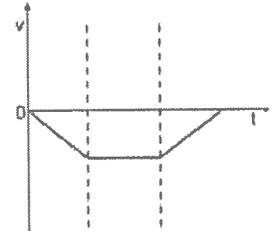
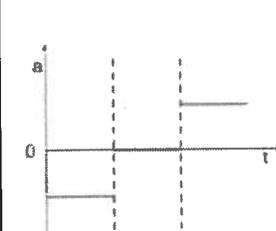
In the next two situations, assume that the ball does not experience any change in velocity while it is on the horizontal part of the ramp.

 <p>For the first 25 units of position, this is just like the very first example we had. After that, we have a case of CVPM, where the slope will be constant and positive, having exactly the value it did at the bottom of the ramp.</p>	<p><b>Math expression for <math>x</math> vs. <math>t</math>:</b></p>	
 <p>The</p>	<p><b>Math expression for <math>v</math> vs. <math>t</math>:</b></p>	
	<p><b>Before 25</b> We learned that the shape in the graph has a general equation in which the vertical axis variable depends on the square of the horizontal axis variable, so <math>x = at^2</math>. But our analysis has shown us that the constant is half the value of the acceleration, so <math>x = \left(\frac{a}{2}\right)t^2</math></p>	<p><b>After 25</b> In this segment of motion, the object is moving at a constant speed. The equation that represents CVPM is <math>x = x_0 + v\Delta t</math></p> <p><b>After 25</b> Here, the velocity is constant, because neither the speed or the direction of the object's motion are changing, so</p>

<p>velocity is always positive, because the slope of <math>x</math> vs. <math>t</math> is always positive. At first it is increasing steadily over time, but at 25 it becomes constant.</p>	<p>acceleration, so <math>v = at</math></p>	<p><math>v = k</math></p>
 <p>Before 25, the <math>v</math> vs. <math>t</math> graph shows a constant, positive slope, so the <math>a</math> vs. <math>t</math> graph should be a horizontal line above zero. Since there is no change in velocity after 25, the acceleration is zero.</p>	<p><b>Math expression for <math>a</math> vs. <math>t</math>:</b></p>	
	<p><b>Before 25</b> Here the <math>v</math> vs. <math>t</math> graph shows a constant, positive slope, so <math>a = k</math></p>	<p><b>After 25</b> Since there is no change in velocity here, the acceleration is zero: <math>a = 0</math></p>
<p><b>Motion map:</b> Because the object is speeding up before 25, we should see dots that are increasingly spaced out with velocity vectors in the forward direction that are getting longer. After 25 the velocity vectors should be the same length, and the dot spacing should be the same, because the object is moving at constant speed. There is constant positive acceleration before 25, so at those dots we see acceleration vectors of the same length pointing in the forward direction.</p>		
		



 <p>Before 30, the object will be speeding up going backwards. This should show increasing steepness with a negative slope. Between 30 and 20, the object is moving at constant speed, but still going backwards. It should show a straight line with a negative slope. From 20 to 0, the object will be slowing down and still going backwards. This should show decreasing steepness and a negative slope.</p>	<b>Math expression for x vs. t:</b>		
	<p><b>50 → 30</b>The starting position wasn't zero, so we need to add the starting position to the change:  <math display="block">x = x_0 + \left(\frac{a}{2}\right) t^2</math></p>	<p><b>30 → 20</b>Here we have constant velocity motion what time does not start at zero, so we need to replace <math>t</math> with <math>\Delta t</math>:  <math display="block">x = x_0 + v\Delta t</math></p>	<p><b>20 → 0</b>The starting position wasn't zero, so we need to add the starting position to the change. Also the starting velocity wasn't zero either, so we need to add the <math>v\Delta t</math> term:  <math display="block">x = x_0 + v\Delta t + \left(\frac{a}{2}\right) \Delta t^2</math></p>
<b>Math expression for v vs. t:</b>			

 <p>The velocity is negative throughout the entire motion. From 50 to 30, the object is speeding up, so the velocity gets farther from the zero line. From 30 to 20, velocity is constant so the velocity graph shows a horizontal line. From 20 to 0 the speed is decreasing, so the velocity line approaches zero.</p>	<p><b>50 → 30</b> This is the equation of a line, where the vertical intercept is 0. We have defined the slope of the v vs. t graph is the acceleration, so <math>v = at</math></p>	<p><b>30 → 20</b> Here, the velocity is constant, because neither the speed or the direction of the object's motion are changing, so <math>v = k</math></p>	<p><b>20 → 0</b> This is the equation of a line, where the vertical intercept is not 0, and time does not start at 0, so <math>v = v_0 + a\Delta t</math></p>
 <p>From 50 → 30 the v vs. t graph shows a constant, negative slope, so the a vs. t graph should be a horizontal line below zero. Since there is no change in velocity from 30 → 20, the acceleration is zero. From 20 → 0</p>	<p><b>Math expression for a vs. t:</b></p>		
<p><b>50 → 30</b> Here the v vs. t graph shows a constant, negative slope, so <math>a = k</math></p>	<p><b>30 → 20</b> Since there is no change in velocity here, the acceleration is zero: <math>a = 0</math></p>	<p><b>20 → 0</b> Here the v vs. t graph shows a constant, positive slope, so <math>a = k</math></p>	

the v vs. t graph shows a constant, positive slope, so the a vs. t graph should be a horizontal line above zero.

**Motion map:** From 50 to 30, the object is speeding up, so the dots are increasingly farther spaced from each other and the velocity vectors are getting longer. Also during this time the acceleration vectors are in the same direction as motion. From 30 to 20, the object is moving at constant speed so the dots are equally spaced, there are no acceleration vectors and the velocity vectors are the same length. From 20 to 0 the object is slowing down, so the dots are closer spaced as time goes on, the velocity vectors decrease in length, and the acceleration vectors are the opposite of the direction of motion.



## CAPM Practice and Exploration #2a

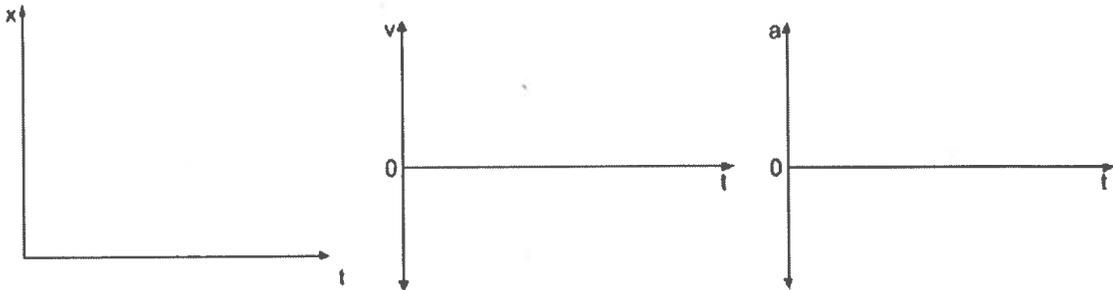
Name \_\_\_\_\_

While cruising along a dark stretch of highway with the cruise control set at 25 m/s, you see, at the fringes of your headlights, that a bridge has been washed out. You apply the brakes and come to a stop in 4.0s. Assume the clock starts the instant you hit the brakes.

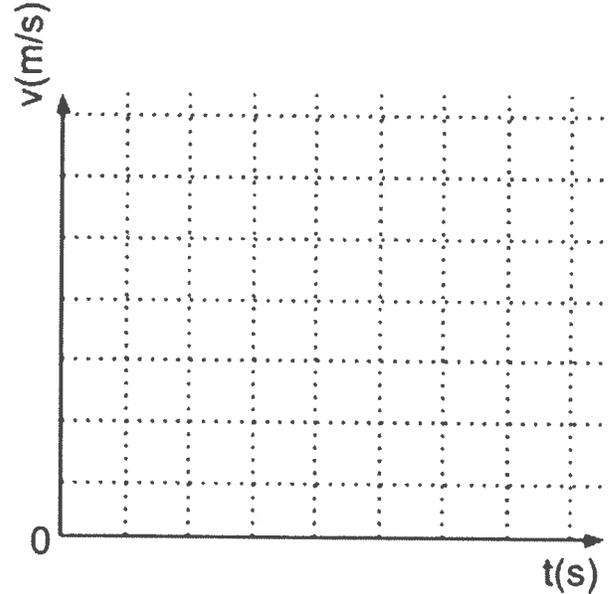
- Construct a qualitative (that means not numerically precise) motion map that represents the motion described above, including position, velocity, and acceleration. Clearly demonstrate how you can determine the direction (sign) of the acceleration from the motion map representation.



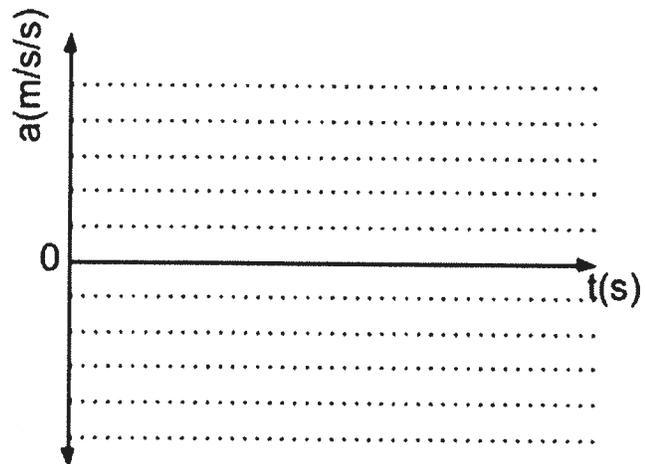
- Sketch some qualitative graphical representations of the situation described above on the axes below:



- Now, construct a quantitatively accurate  $v$  vs.  $t$  graph to describe the situation.
- On the  $v$  vs.  $t$  graph that you just made, graphically represent the car's displacement during braking.
- Using the graphical representation, determine how far the car traveled during braking. (Make sure you can explain how you arrived at your answer.)



- In order to draw the  $a$  vs.  $t$  graph, you need to know the car's acceleration. Please do this, then sketch a quantitatively accurate  $a$  vs.  $t$  graph.



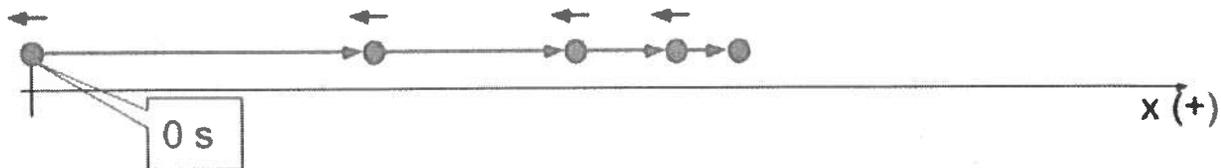
- Using the time-dependent kinematics equation, determine how far the car traveled during braking. How does this compare with the answer you arrived at graphically.

## CAPM Practice and Exploration #2a Solutions

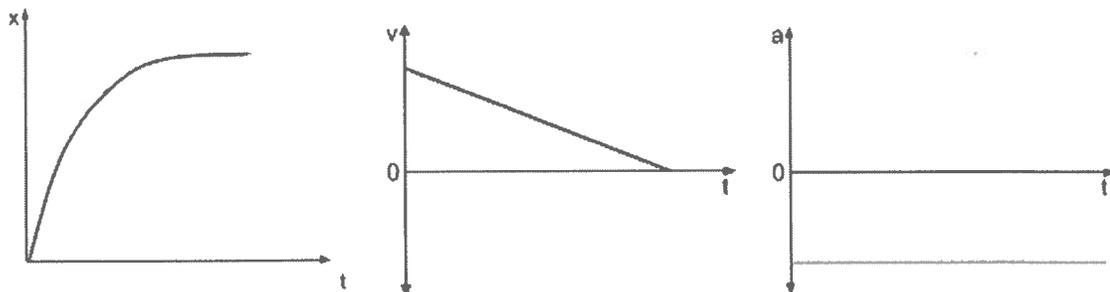
Name \_\_\_\_\_

While cruising along a dark stretch of highway with the cruise control set at 25 m/s, you see, at the fringes of your headlights, that a bridge has been washed out. You apply the brakes and come to a stop in 4.0s. Assume the clock starts the instant you hit the brakes.

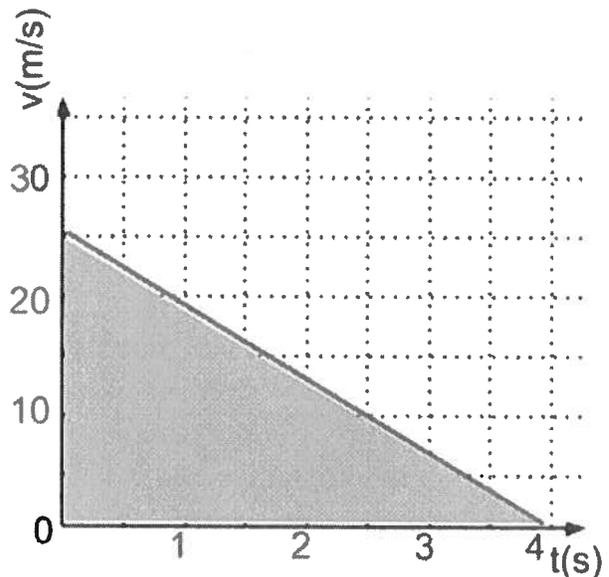
- Construct a qualitative (that means not numerically precise) motion map that represents the motion described above, including position, velocity, and acceleration. Clearly demonstrate how you can determine the direction (sign) of the acceleration from the motion map representation. Because the object is moving forward, the velocity vectors point in the positive direction. Because the object is slowing down, the dots are spaced closer together with increasing time. Also because the object is slowing down, the acceleration vectors are opposite the direction of the velocity vectors.



- Sketch qualitative graphical representations of the situation described above on the axes below:
  - $x$  increases because the car is moving forward. But the steepness of the line is decreasing to zero, because the car is slowing down.
  - Velocity is positive at the beginning, because the car is moving forward. But velocity reaches zero as time goes on, because the car slows to a stop.
  - Because the slope of  $v$  vs. time is negative, acceleration is negative. Because the  $v$  vs. time graph is a straight line, acceleration is constant.



- Now, construct a quantitatively accurate  $v$  vs.  $t$  graph to describe the situation. At the start the velocity is 25 m/s. After 4 seconds, the velocity is zero. I'm assuming that the change in velocity is constant.



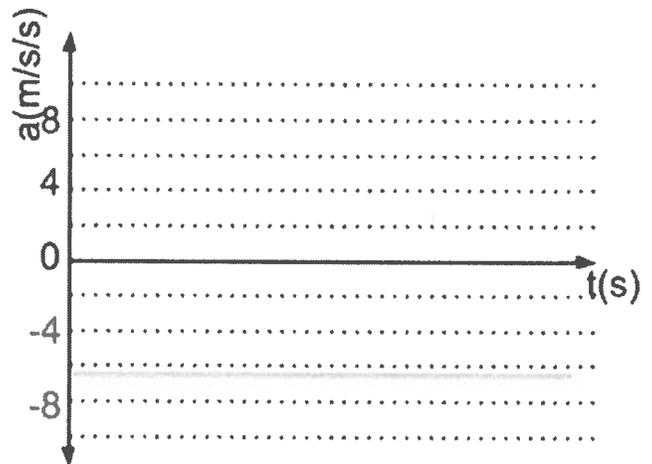
- On the  $v$  vs.  $t$  graph that you just made, graphically represent the car's displacement during braking. Displacement is the area under the velocity versus time graph.

- Using the graphical representation, determine how far the car traveled during braking. (Make sure you can explain how you arrived at your answer.)

$$\Delta x = \frac{25 \frac{m}{s} \cdot 4 s}{2} = \frac{100 m}{2} = 50 m$$

- In order to draw the  $a$  vs.  $t$  graph, you need to know the car's acceleration. Please do this, then sketch a quantitatively accurate  $a$  vs.  $t$  graph.

$$a = \frac{\Delta v}{\Delta t} = \frac{-25 \frac{m}{s}}{4 s} = -6.25 \frac{m}{s}$$



- Using the time-dependent kinematics equation, determine how far the car traveled during braking. How does this compare with the answer you arrived at graphically.

$$\Delta x = v_i \Delta t + \frac{a}{2} \Delta t^2 = \left(25 \frac{m}{s}\right) (4 s) + \left(\frac{-6.25 \frac{m}{s}}{2}\right) (4 s)^2 = 100 m - 50 m = 50 m$$

---

They are the same values. this is probably because we derived the time dependent kinematics equation using a velocity vs. time graph.

## CAPM Practice and Exploration #2b

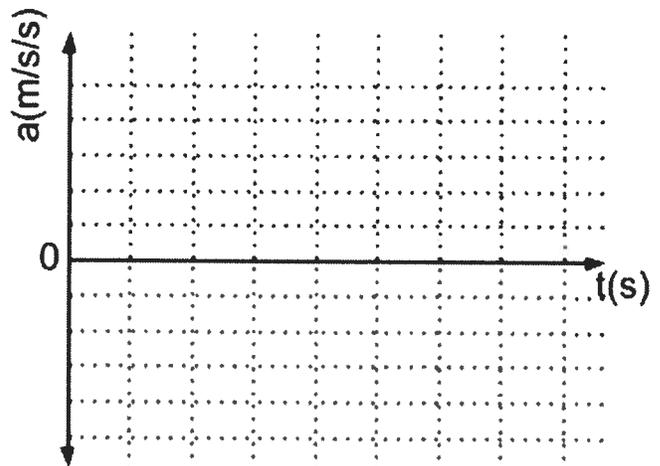
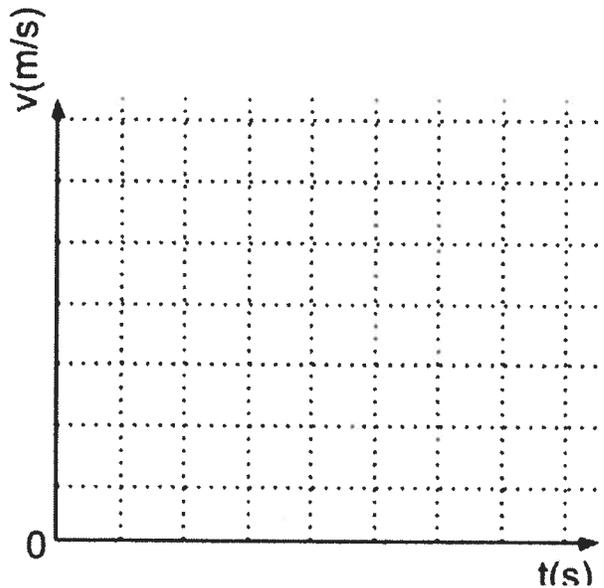
Name \_\_\_\_\_

While cruising along a dark stretch of highway with the cruise control set at 30 m/s, you see, at the fringes of your headlights, some roadkill on the highway. It takes you 0.5 s to react, then you apply the brakes and come to a stop 3.0 s later. Assume the clock starts the instant you see the roadkill.

- Construct a qualitative motion map that represents the motion described above, including position, velocity, and acceleration. Instead of a dot every second, place a dot every  $\frac{1}{2}$  second.



- Construct a quantitatively accurate v vs. t graph to describe the situation.
- On the v vs. t graph that you just made, graphically represent the car's displacement during braking.
- Using the graphical representation, determine how far the car traveled during braking. (Make sure you can explain how you arrived at your answer.)
- In order to draw the a vs. t graph, you need to know the car's acceleration. Please do this, then sketch a quantitatively accurate a vs. t graph.



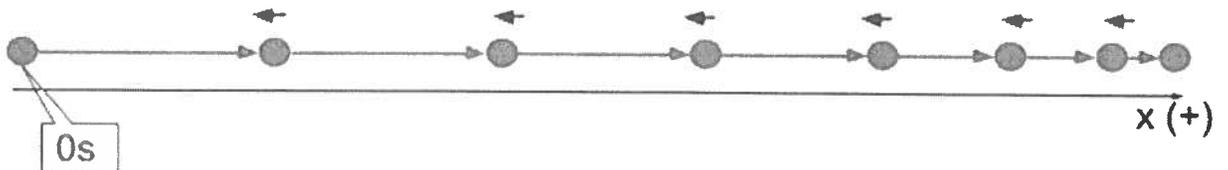
- Two kinds of motion occur in this case:
  - For the first 0.5s, the car's motion can be modeled as CVPM. Use the time-dependent kinematics equation to find the displacement of the car during this time.
  
  - For the remainder of the time, the car's motion can be modeled as CAPM. Use the time-dependent kinematics equation to find the displacement of the car during this time.
  
  - Sum the two displacements to find the overall displacement for the whole event. How does it compare with the displacement you found graphically?
  
  - Explain why  $\Delta x = v_i \Delta t$  (from CVPM) and  $\Delta x = v_i \Delta t + \frac{a}{2} \Delta t^2$  (from CAPM) are equivalent expressions.

## CAPM Practice and Exploration #2b Solutions

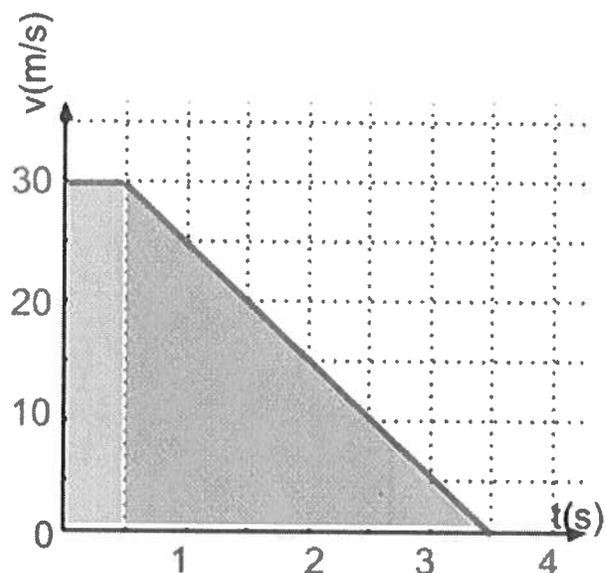
Name \_\_\_\_\_

While cruising along a dark stretch of highway with the cruise control set at 30 m/s, you see, at the fringes of your headlights, some roadkill on the highway. It takes you 0.5 s to react, then you apply the brakes and come to a stop 3.0 s later. Assume the clock starts the instant you see the roadkill.

- Construct a qualitative motion map that represents the motion described above, including position, velocity, and acceleration. Instead of a dot every second, place a dot every  $\frac{1}{2}$  second. If the positive direction is the direction the car was moving in the first place, the first half second, the velocity is positive and constant, so there is no acceleration vector and the velocity vector points in the positive direction. After that, the car is slowing down and moving in the positive direction, so the velocity vectors continue to point forward, but get shorter as the dot spacing gets smaller. The acceleration vectors are opposite the direction of motion.



- Construct a quantitatively accurate  $v$  vs.  $t$  graph to describe the situation. The velocity is a constant 30 m/s for the first 0.5 s. From then until 3.5 s, the velocity constantly decreases to 0.
- On the  $v$  vs.  $t$  graph that you just made, graphically represent the car's displacement during braking. Displacement is always the area under the graph of  $v$  vs.  $t$ . The CVPM part is shaded blue and the CAPM part is shaded pink.



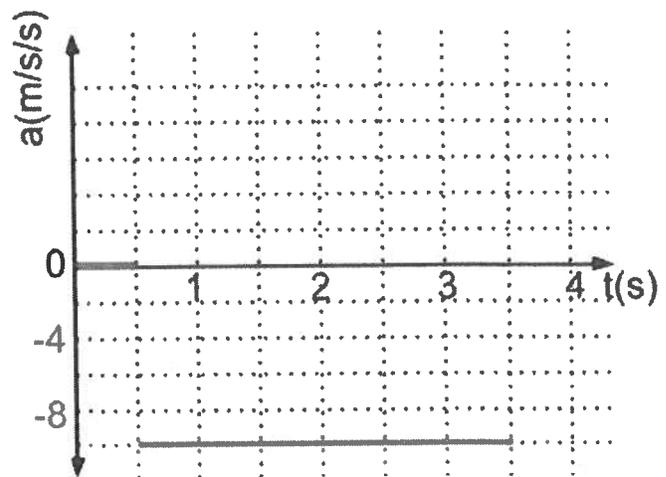
- Using the graphical representation, determine how far the car traveled during braking. (Make sure you can explain how you arrived at your answer.)

- CVPM part: 
$$\Delta x = \left( \frac{30 \text{ m}}{\text{s}} \right) (0.5 \text{ s}) = 15 \text{ m}$$

- CAPM part: 
$$\Delta x = \frac{\left( \frac{30 \text{ m}}{\text{s}} \right) (3.0 \text{ s})}{2} = 45 \text{ m}$$

- So in total  $\Delta x = 15 \text{ m} + 45 \text{ m} = 60 \text{ m}$

- In order to draw the a vs. t graph, you need to know the car's acceleration. Please do this, then sketch a quantitatively accurate a vs. t graph



$$a = \frac{\Delta v}{\Delta t} = \frac{-30 \frac{\text{m}}{\text{s}}}{3.0 \text{ s}} = -10 \frac{\text{m}}{\text{s}^2}$$

- Two kinds of motion occur in this case:
  - For the first 0.5s, the car's motion can be modeled as CVPM. Use the time-dependent kinematics equation to find the displacement of the car during this time.

$$\Delta x = v_i \Delta t + \frac{a}{2} \Delta t^2 = \left( \frac{30 \text{ m}}{\text{s}} \right) (0.5 \text{ s}) + \frac{0}{2} (0.5 \text{ s})^2 = 15 \text{ m}$$

- For the remainder of the time, the car's motion can be modeled as CAPM. Use the time-dependent kinematics equation to find the displacement of the car during this time.

$$\Delta x = \left( \frac{30 \text{ m}}{\text{s}} \right) (3.0 \text{ s}) + \frac{-10 \frac{\text{m}}{\text{s}^2}}{2} (3.0 \text{ s})^2 = 45 \text{ m}$$

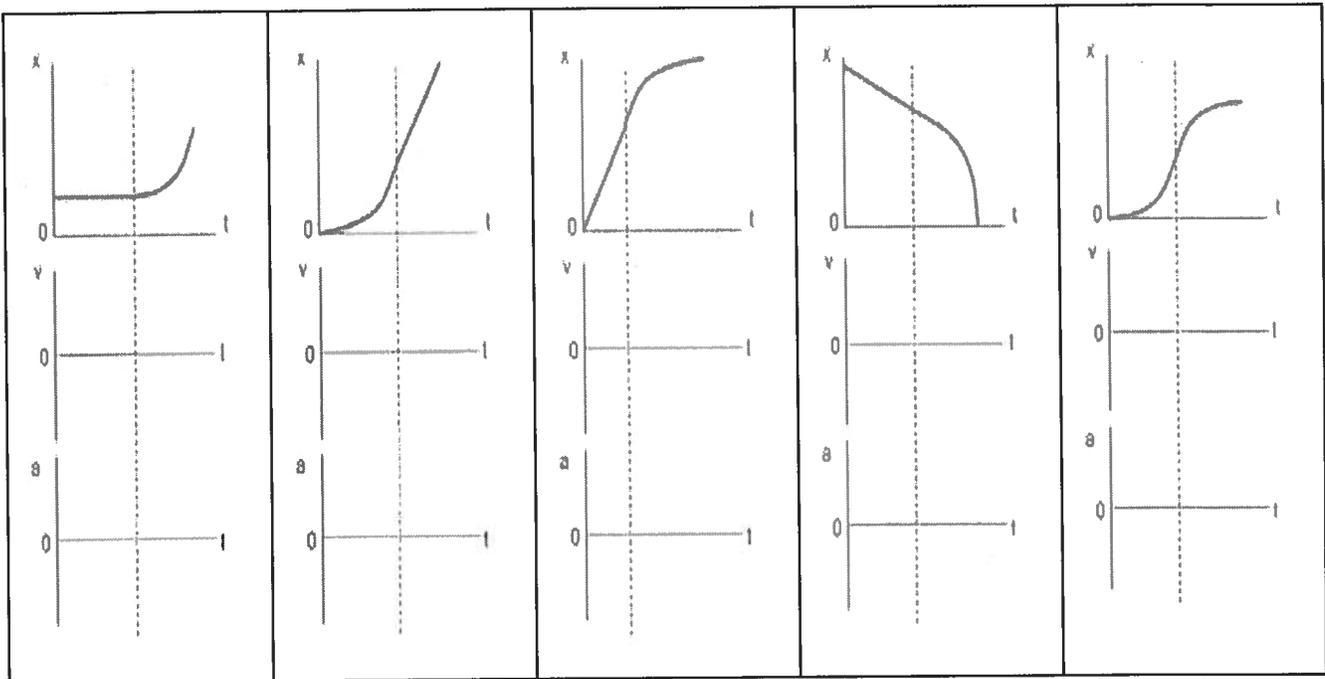
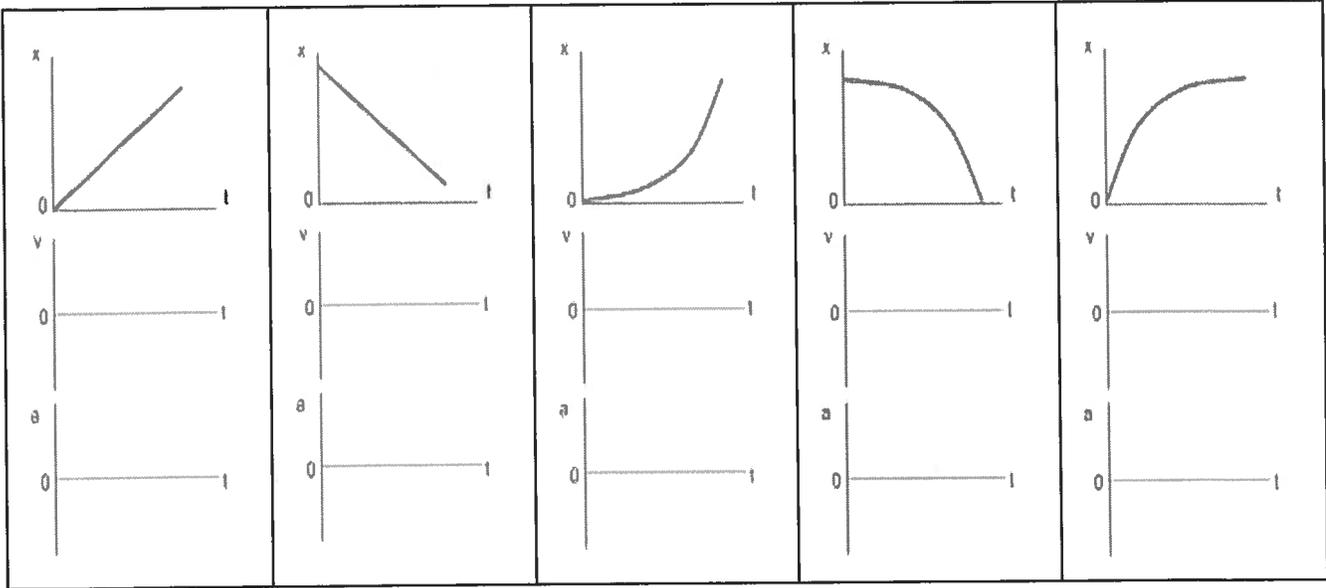
- Sum the two displacements to find the overall displacement for the whole event. How does it compare with the displacement you found graphically?

$$\Delta x = 15 \text{ m} + 45 \text{ m} = 60 \text{ m} \text{ It's the same value}$$

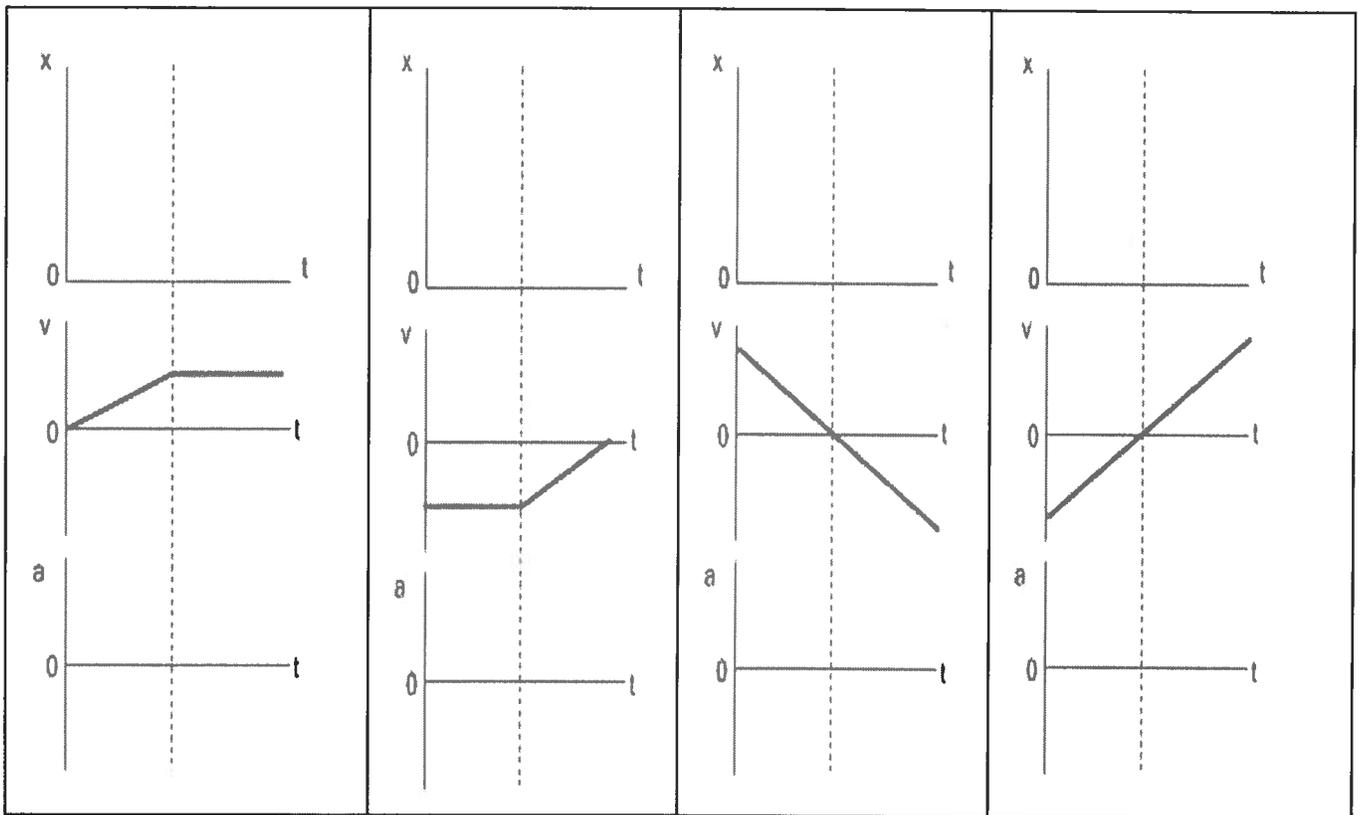
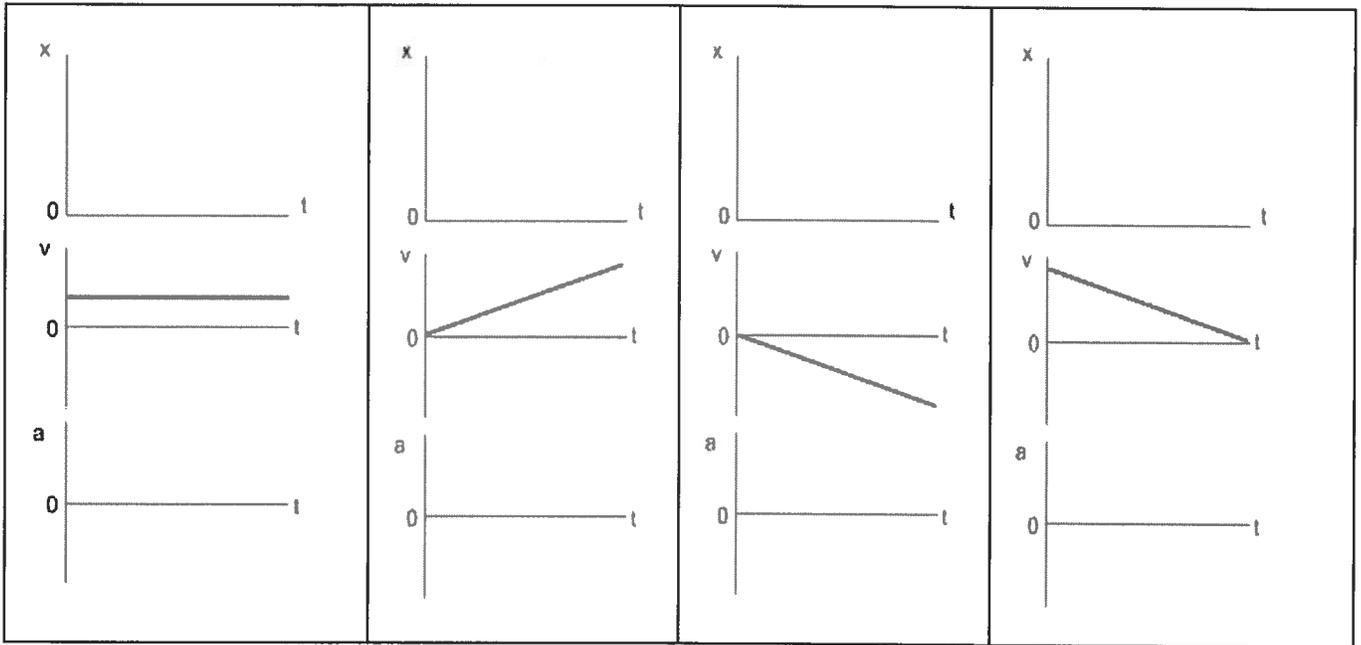
- Explain why  $\Delta x = v \Delta t$  (from CVPM) and  $\Delta x = v_i \Delta t + \frac{a}{2} \Delta t^2$  (from CAPM) are equivalent expressions. Because in CVPM  $a = 0$ , so the second term is zero. Also there is no change in velocity, so  $v_i$  is just  $v$ .

## CAPM Stacks of Kinematics Curves

*Use the  $x$  vs  $t$  graphs to sketch  $v$  vs.  $t$  and  $a$  vs.  $t$  graphs that model the same motion:*

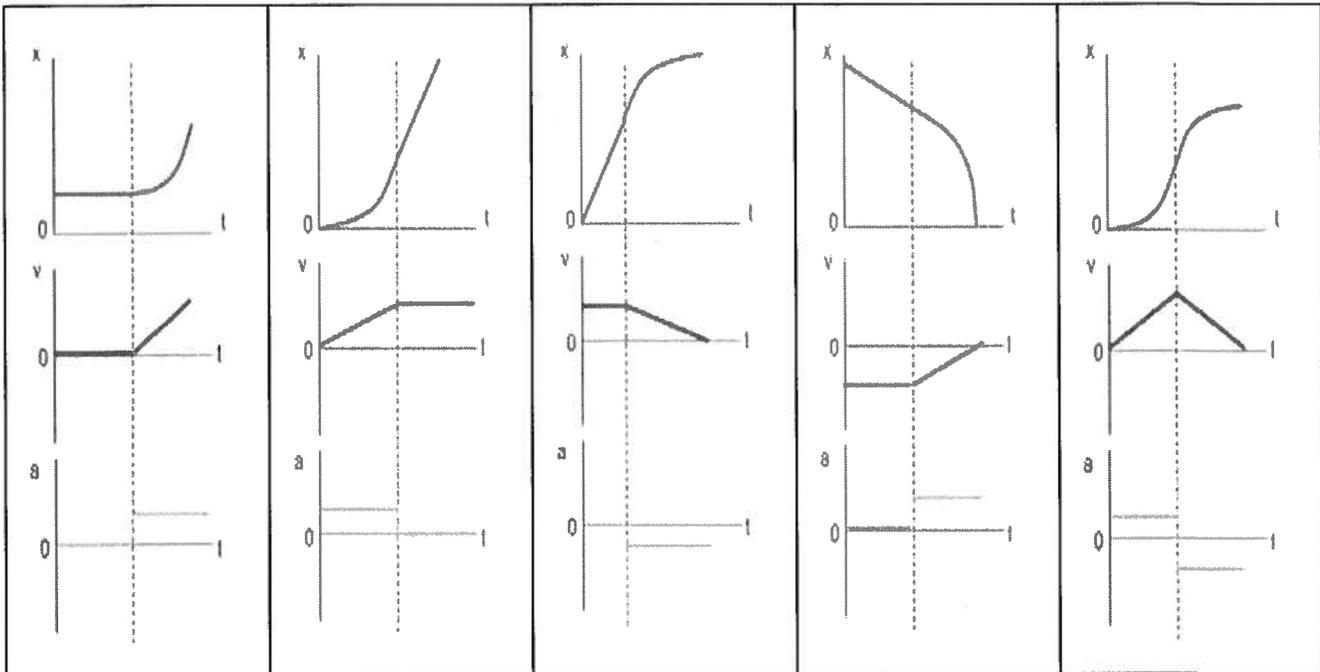
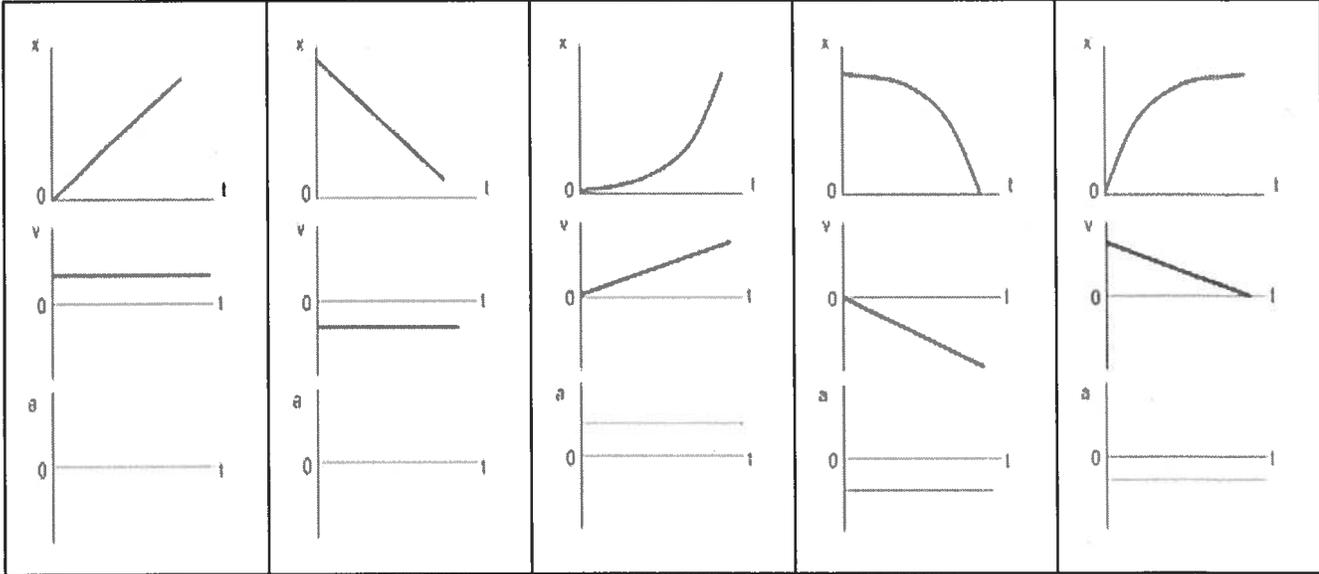


Use the  $v$  vs  $t$  graphs to sketch  $x$  vs.  $t$  and  $a$  vs.  $t$  graphs that model the same motion:

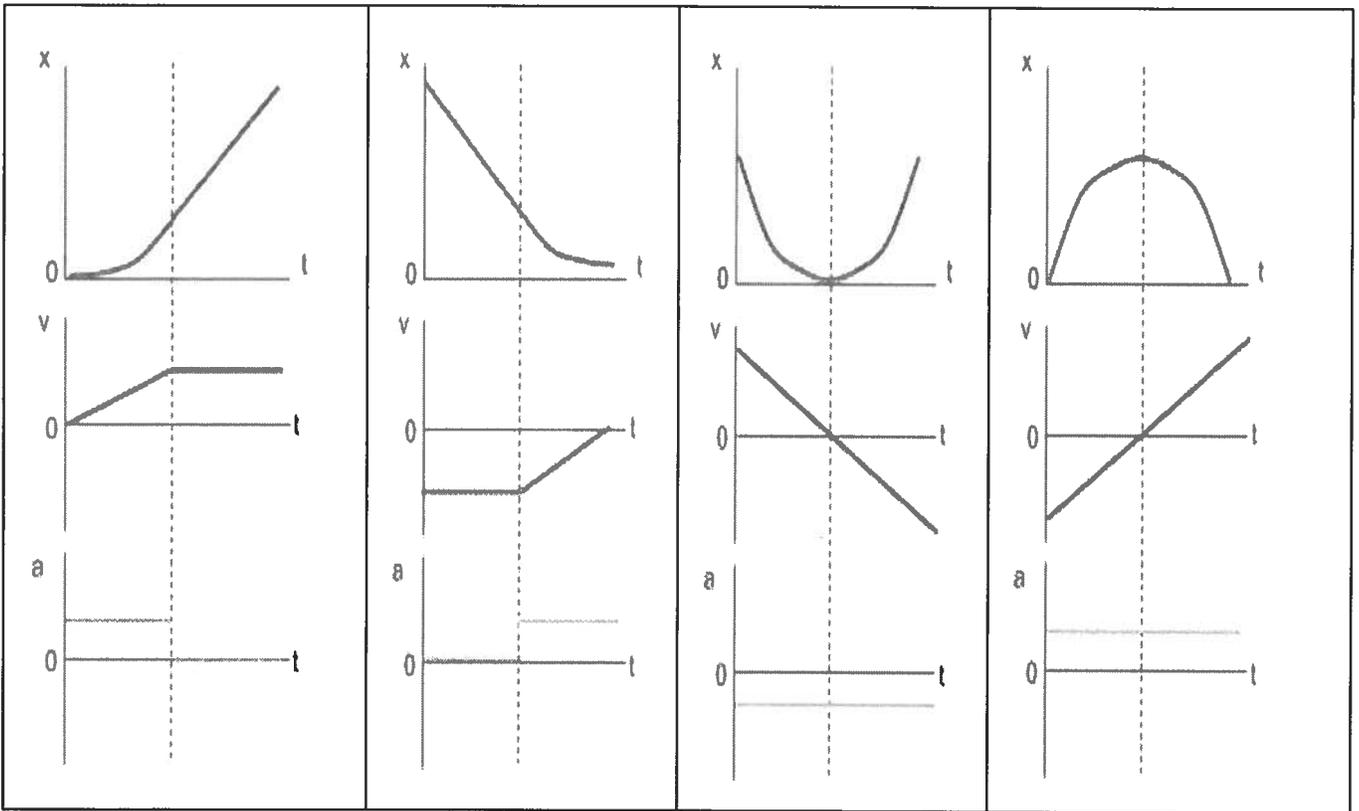
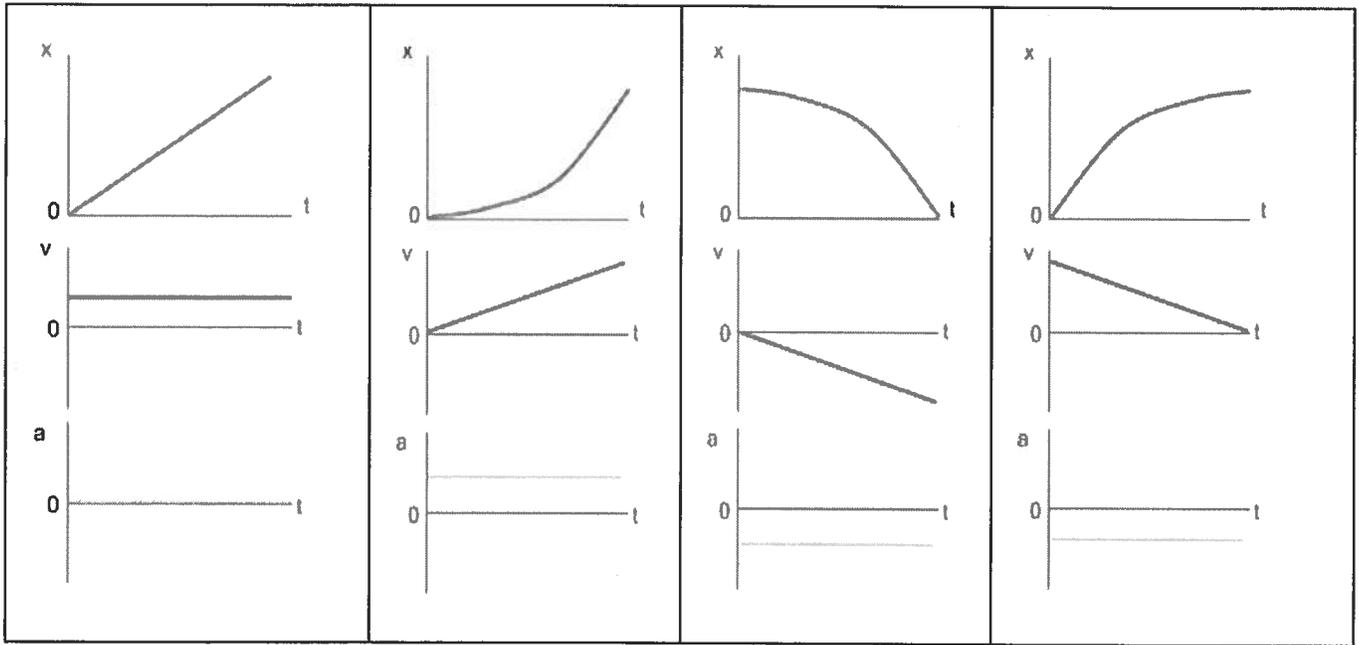


## CAPM Stacks of Kinematics Curves Solutions

*Use the  $x$  vs  $t$  graphs to sketch  $v$  vs.  $t$  and  $a$  vs.  $t$  graphs that model the same motion:*

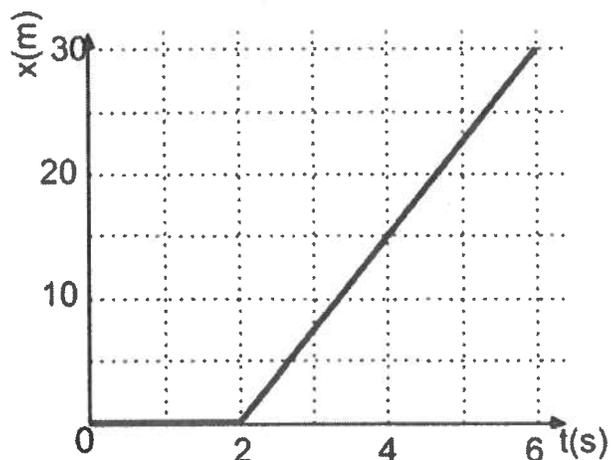


Use the  $v$  vs  $t$  graphs to sketch  $x$  vs.  $t$  and  $a$  vs.  $t$  graphs that model the same motion:



### CAPM Practice and Exploration #3

Name \_\_\_\_\_



- Describe in words the motion of the object from 0 - 6.0 s.

- Construct a qualitative motion map to describe the motion of the object.

\_\_\_\_\_ x (+)

- Determine the instantaneous velocity at each second of the motion:

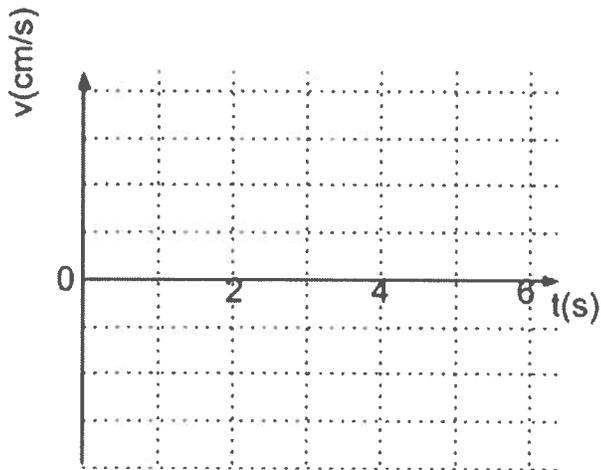
time (s)	0	1	2	3	4	5	6
v (m/s)							

- What is the simple average of all these velocities?
- What is the average velocity from 0 s to 6 s using the definition.

$$v_{av} \equiv \frac{\Delta x}{\Delta t}$$

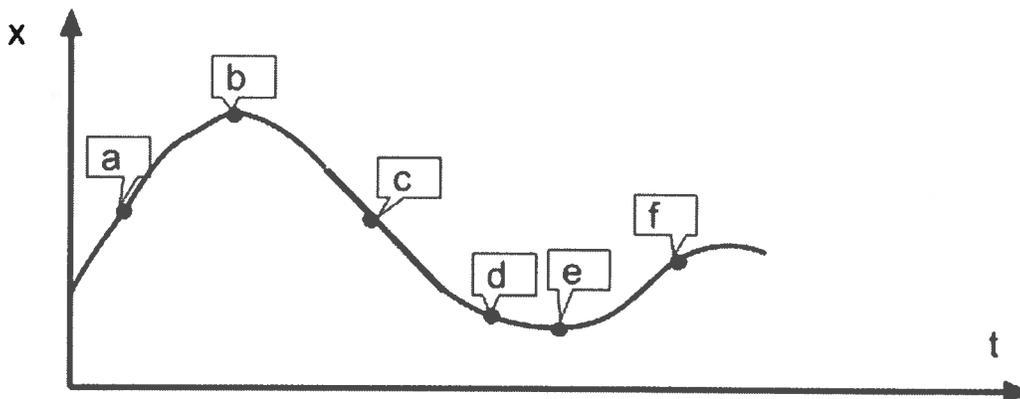
- Which of these numbers provides more meaningful information about the motion of the object?

- Graphically represent the relationship between velocity and time for the object described above.



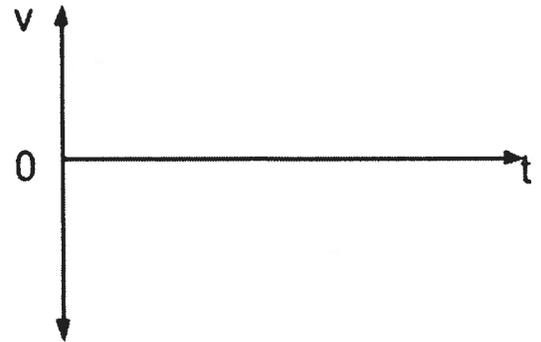
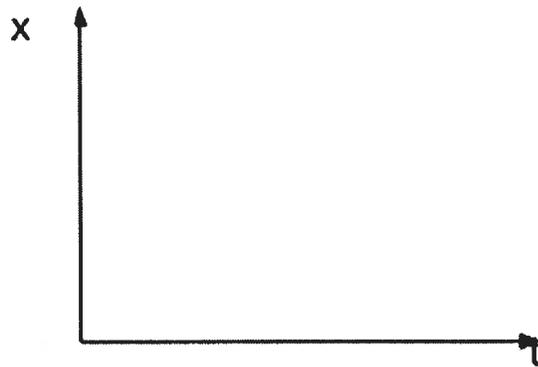
- From your  $v$  vs.  $t$  graph, determine the total displacement of the object.

- This graph represents the motion of an object.

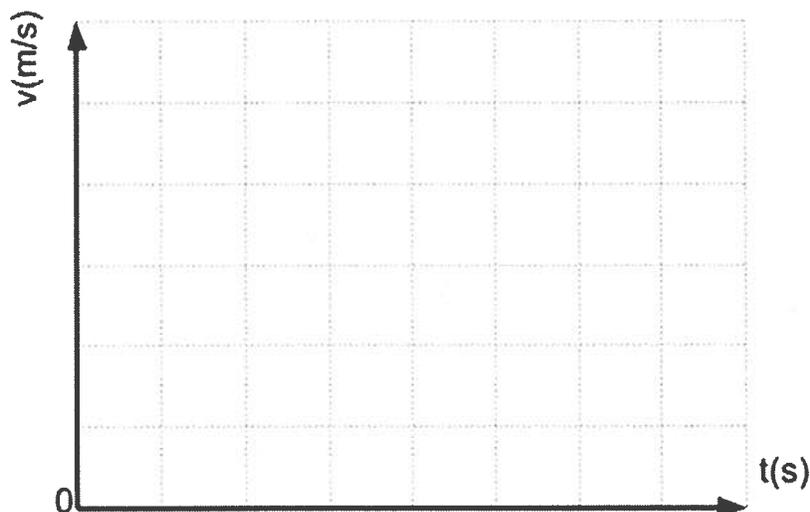


- At what point(s) on the graph is the object moving most slowly? How do you know?
- Over what interval(s) on the graph is the object speeding up? How do you know?
- Over what interval(s) on the graph is the object slowing down? How do you know?
- At what point(s) on the graph is the object changing direction? How do you know?

- A stunt car driver testing the use of air bags drives a car at a constant speed of 25 m/s for a total of 100. m. He applies his brakes and accelerates uniformly to a stop just as he reaches a wall 50. m away.
  - Sketch qualitative graphical representations of his motion:



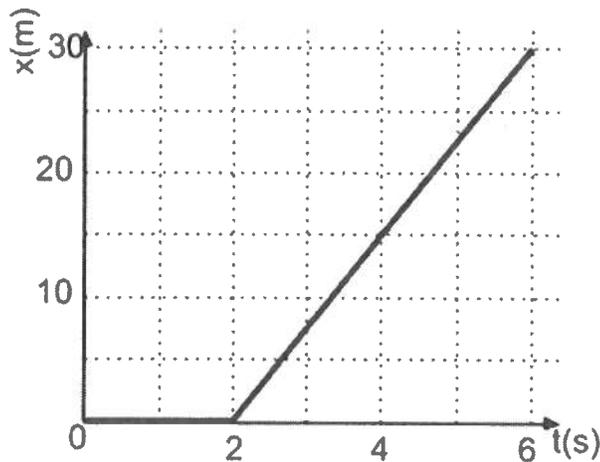
- How long does it take for the car to travel the first 100 m?
- Remember that the area under a velocity versus time graph equals the displacement. How long must the brakes be applied for the car to come to a stop in 50 m?
- Now that you know the total time of travel, sketch a quantitative velocity versus time graph.



- What acceleration is provided by the brakes? How do you know?

## CAPM Practice and Exploration #3 Solutions

Name \_\_\_\_\_



- Describe in words the motion of the object from 0 - 6.0 s. Because the position doesn't change for the first two seconds, the object is not moving during that time. Because the position's change is modeled by a straight line

with a positive slope after 2 s, the object is moving forward at a constant speed after 2 seconds.

- Construct a qualitative motion map to describe the motion of the object. Because the object stays at position 0 for two seconds, I'll stack dots at that position. After that, the object moves forward at a constant speed, so I'll show equal dot spacing and equal-length velocity vectors pointing forward.



- Determine the instantaneous velocity at each second of the motion: Instantaneous velocity is the slope of the x vs t line at that point. For the first 2 seconds, the line is flat so the velocity is 0. After that, the slope is unchanging

and velocity is 
$$v = \frac{\Delta x}{\Delta t} = \frac{30 \text{ m}}{4 \text{ s}} = 7.5 \frac{\text{m}}{\text{s}}$$

time (s)	0	1	2	3	4	5	6
v (m/s)	0	0	7.5	7.5	7.5	7.5	7.5

- What is the simple average of all these velocities?

$$\frac{0 + 0 + 7.5 + 7.5 + 7.5 + 7.5 + 7.5}{7} = 5.36 \frac{m}{s}$$

- What is the average velocity from 0 s to 6 s using the definition.

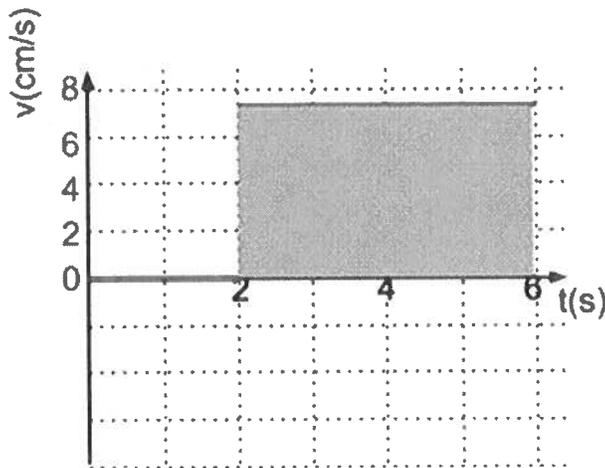
$$v_{av} \equiv \frac{\Delta x}{\Delta t} = \frac{30 \text{ m}}{6 \text{ s}} = 5 \frac{m}{s}$$

- Which of these numbers provides more meaningful information about the motion of the object? If you told someone the simple average and they used it to calculate the position of the object at 6 s, they would get

$$\Delta x = v \Delta t = \left( \frac{5.36 \text{ m}}{s} \right) (6.0 \text{ s}) = 32.2 \text{ m}$$

, which is wrong. So the value using the physics definition is the better one.

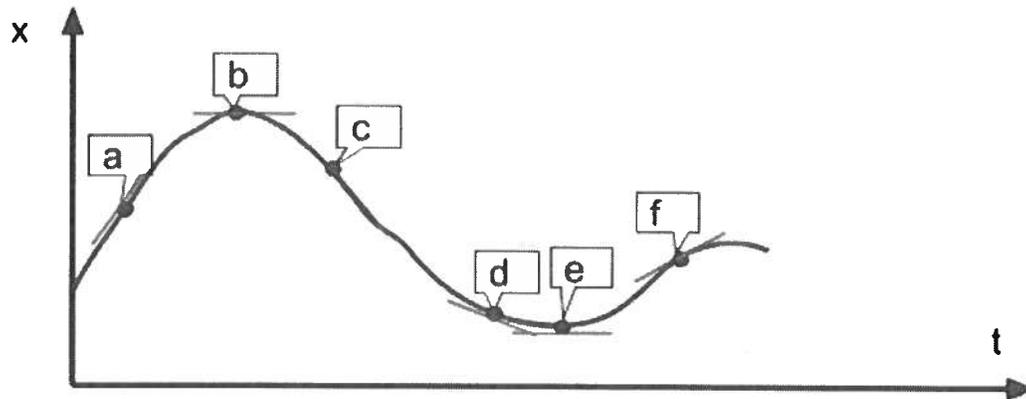
- Graphically represent the relationship between velocity and time for the object described above. This just plots the velocity values already calculated.



$$\Delta x = 7.5 \frac{m}{s} \cdot 4 \text{ s} = 30 \text{ m}$$

- From your  $v$  vs.  $t$  graph, determine the total displacement of the object. Displacement is the area under the graph of velocity vs. time, shaded here in pink.

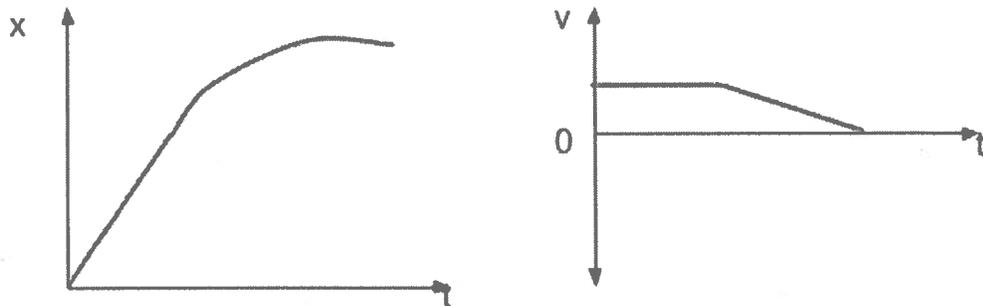
- This graph represents the motion of an object.



- At what point(s) on the graph is the object moving most slowly? How do you know? Objects are moving slower when the steepness of the slope of the tangent line is lowest. I've put tangent lines on the graph for reference. The lowest it can be is zero, so that makes b and e the points where the object is moving slowest.
- Over what interval(s) on the graph is the object speeding up? How do you know? Objects are speeding up when the slope of the tangent line is becoming more steep. This would be from  $b \rightarrow c$  and from  $e \rightarrow f$ .
- Over what interval(s) on the graph is the object slowing down? How do you know? Objects are slowing down when the slope of the tangent line is becoming less steep. This would be from  $a \rightarrow b$ , from  $c \rightarrow d$  and from  $d \rightarrow e$ .
- At what point(s) on the graph is the object changing direction? How do you know? Objects are changing direction when the slope of the tangent line changes sign. This happens at b and e .

- A stunt car driver testing the use of air bags drives a car at a constant speed of 25 m/s for a total of 100. m. He applies his brakes and accelerates uniformly to a stop just as he reaches a wall 50. m away.

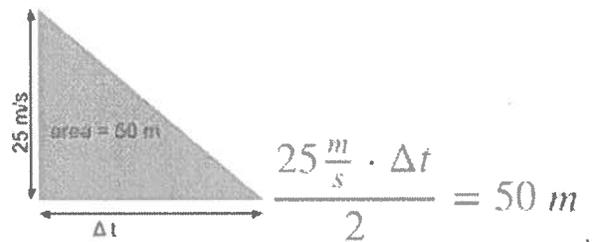
- Sketch qualitative graphical representations of his motion: x vs. t: The car is moving forward so the slope is generally positive. At first, the slope is constant because the car is moving at constant speed. After reaching 100 m, the car is slowing down, so at that point, the slope should become less steep until it is zero. v vs. t: The car has a constant positive velocity for a while, then it stays positive, but approaches 0 as it slows to a stop.



- How long does it take for the car to travel the first 100 m? Since the car is moving at a constant velocity,  $\Delta x = v\Delta t$ , so

$$\Delta t = \frac{\Delta x}{v} = \frac{100 \text{ m}}{25 \frac{\text{m}}{\text{s}}} = 4 \text{ s}$$

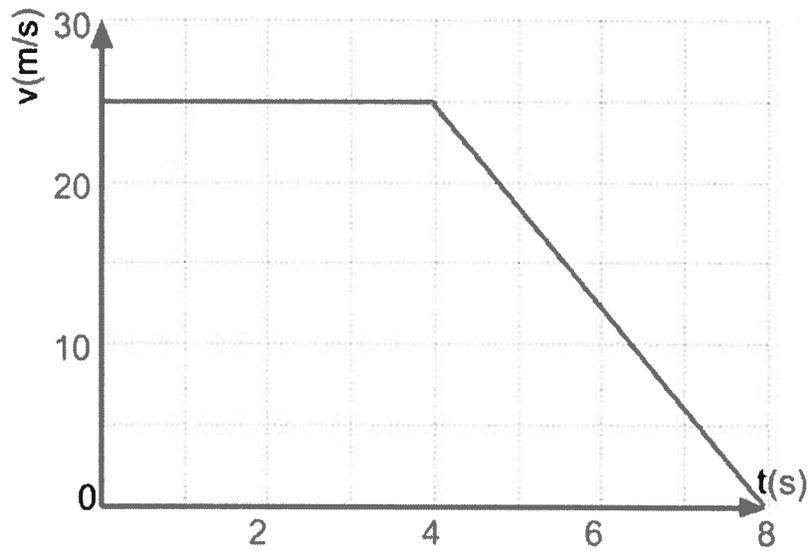
- Remember that the area under a velocity versus time graph equals the displacement. How long must the brakes be applied for the car to come to a stop in 50 m? If I look at the last section of the v vs. t graph, knowing that the area of that triangle is the displacement (50 m) and that the height of the triangle is the starting velocity (25 m/s) I can find the missing base



of the triangle, which is the  $\Delta t$ .

so  $25 \frac{\text{m}}{\text{s}} \cdot \Delta t = 100 \text{ m}$ , so  $\Delta t = \frac{100 \text{ m}}{25 \frac{\text{m}}{\text{s}}} = 4 \text{ s}$

- Now that you know the total time of travel, sketch a quantitative velocity versus time graph. Here I'm just graphing the results I found above.



- What acceleration is provided by the brakes? How do you know? The change in velocity only happens in the last 4 seconds, so

$$a = \frac{\Delta v}{\Delta t} = \frac{-25 \frac{\text{m}}{\text{s}}}{4 \text{ s}} = -6.25 \frac{\text{m}}{\text{s}}$$

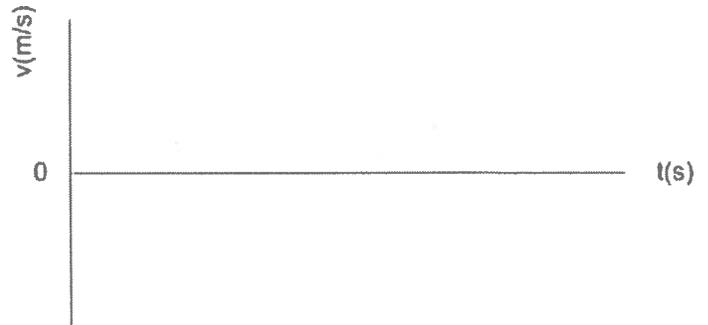
## CAPM Practice and Exploration #4

Name \_\_\_\_\_

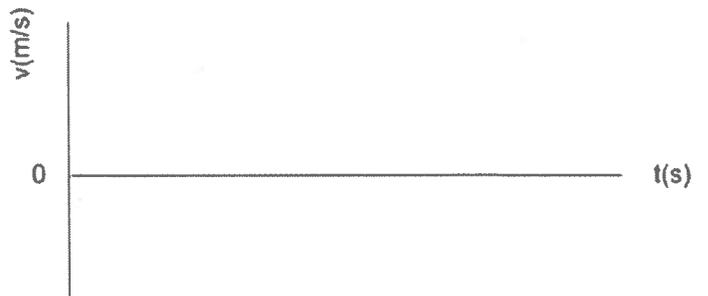
1. A poorly tuned Geo Metro can accelerate from rest to a speed of 28 m/s in 20 s.

a. What is the average acceleration of the car?

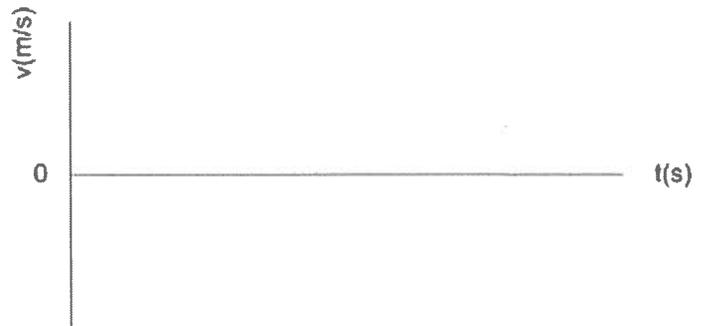
b. What distance does the car travel in this time?



2. At  $t = 0$  a car has a speed of 30 m/s. At  $t = 6\text{s}$ , its speed is 14 m/s. What is its average acceleration during this time interval?



3. A bear spies some honey and takes off from rest accelerating at a rate of 2.0 m/s/s. If the honey is 16 m away, how fast will his snout be going when it reaches the treat?



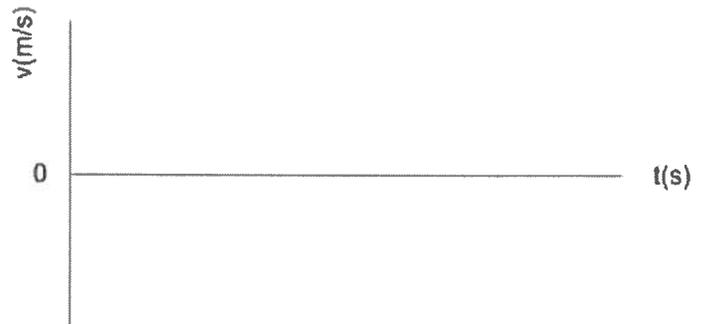
4. A bus moving at 20 m/s (when  $t = 0$ ) slows at a rate of 4 m/s each second.

a. How long does it take the bus to stop?

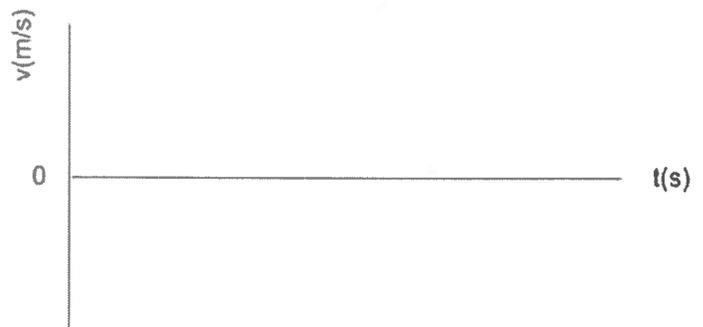
b. How far does it travel while braking?



5. A physics student skis down a hill accelerating at a constant  $2.0 \text{ m/s}^2$ . If it takes her  $15 \text{ s}$  to reach the bottom, how long is the ski hill?

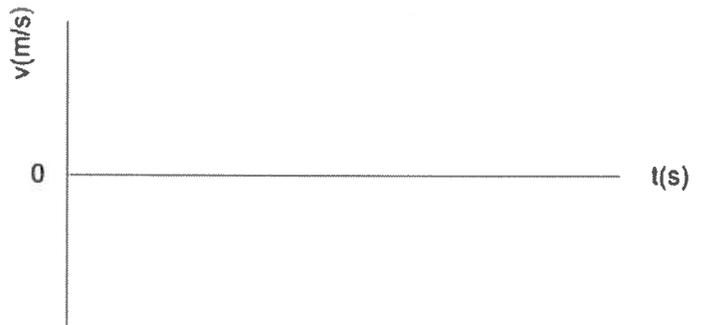


6. A dog runs down his driveway with a starting speed of  $5 \text{ m/s}$  for  $8 \text{ s}$ , then uniformly increases his speed to  $10 \text{ m/s}$  in  $5 \text{ s}$ .
- What was his acceleration during the 2nd part of the motion?

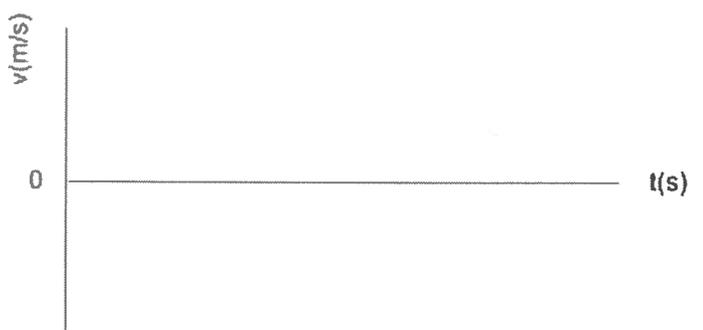


- How far did he run, in total?

7. A mountain goat starts a rock slide and the rocks crash down the mountain  $100 \text{ m}$ . If the rocks reach the bottom in  $5 \text{ s}$ , what was their acceleration?



8. A car whose initial speed is  $30 \text{ m/s}$  slows uniformly to  $10 \text{ m/s}$  in  $5 \text{ s}$ .
- Determine the acceleration of the car.
  - Determine the distance it travels in from  $t = 2 \text{ s}$  to  $t = 3 \text{ s}$ .



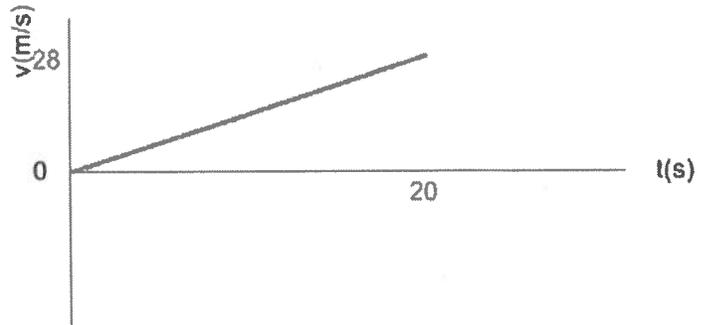
## CAPM Practice and Exploration #4 Solutions

Name \_\_\_\_\_

1. A poorly tuned Geo Metro can accelerate from rest to a speed of 28 m/s in 20 s.

- a. What is the average acceleration of the car?

$$a = \frac{\Delta v}{\Delta t} = \frac{28 \frac{m}{s}}{20 s} = 1.4 \frac{m}{s}$$

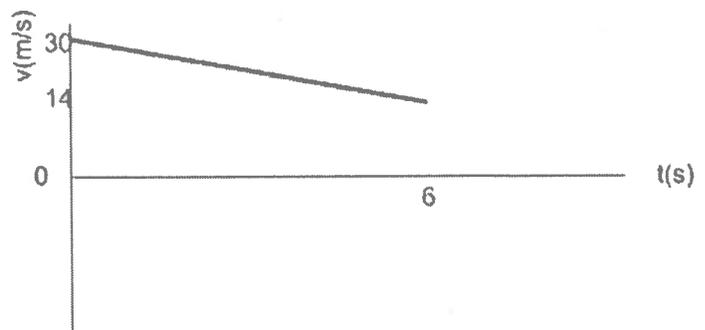


- b. What distance does the car travel in this time?

$$\Delta x = \frac{28 \frac{m}{s} \cdot 20 s}{2} = 280 m$$

2. At  $t = 0$  a car has a speed of 30 m/s. At  $t = 6s$ , its speed is 14 m/s. What is its average acceleration during this time interval?

$$a = \frac{\Delta v}{\Delta t} = \frac{-6 \frac{m}{s}}{6 s} = -1 \frac{m}{s}$$



3. A bear spies some honey and takes off from rest accelerating at a rate of 2.0 m/s/s. If the honey is 16 m away, how fast will his snout be going when it reaches the treat?

Method 1: Guess-n-check:

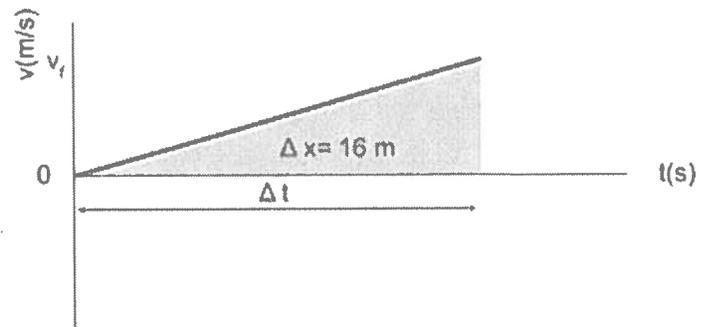
- try  $\Delta t = 1s$ :

$$v = v_i + a\Delta t = 2.0 \frac{m}{s} \cdot 1.0s = 2.0 \frac{m}{s}, \text{ so}$$

$$\Delta x = \frac{2.0 \frac{m}{s} \cdot 1s}{2} = 1.0 m$$

...not far enough so try a longer  $\Delta t$ ...

- eventually you will find that at 4s,  $\Delta x = 16 m$  and  $v = 8 m/s$ .

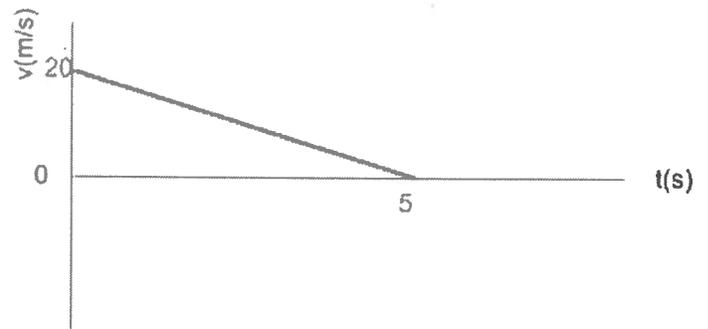


Method 2: time-independent equation:

$$v_f^2 = v_i^2 + 2a\Delta x = 0 + 2 \left( 2.0 \frac{m}{s} \right) (16m) = 64 \frac{m^2}{s^2}, \text{so}$$

$$v_f = \sqrt{64 \frac{m^2}{s^2}} = 8 \frac{m}{s}$$

4. A bus moving at 20 m/s (when  $t = 0$ ) slows at a rate of 4 m/s each second.



- a. How long does it take the bus to stop?  $v = v_i + a\Delta t$ , so

$$\left( 4 \frac{m}{s} \right) \Delta t = 20 \frac{m}{s}, \text{so}$$

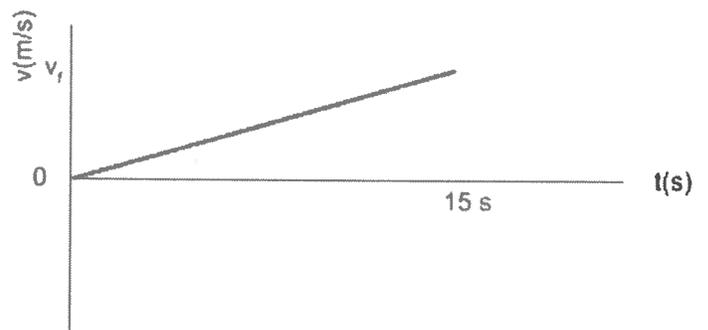
$$\Delta t = 5 \text{ s}$$

- b. How far does it travel while braking?

$$\Delta x = \frac{\left( 20 \frac{m}{s} \right) \cdot (5 \text{ s})}{2} = 50 \text{ m}$$

5. A physics student skis down a hill accelerating at a constant 2.0 m/s/s. If it takes her 15 s to reach the bottom, how long is the ski hill?

Method 1: Two steps:



$$v = v_i + a\Delta t = \left( 2.0 \frac{m}{s} \right) \cdot 15 \text{ s} = 30 \frac{m}{s}$$

$$\Delta x = \frac{\left(30 \frac{m}{s}\right) \cdot (15 s)}{2} = 225 m$$

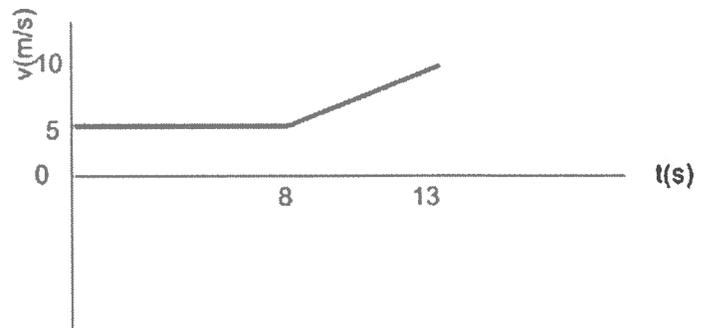
Method 2: time-dependent equation:

$$\Delta x = v_i \cdot \Delta t + \frac{a}{2} \cdot \Delta t^2 = 0 + \left(\frac{2.0 \frac{m}{s^2}}{2}\right) \cdot (15 s)^2 = 225 m$$

6. A dog runs down his driveway with a starting speed of 5 m/s for 8 s, then uniformly increases his speed to 10 m/s in 5 s.

- a. What was his acceleration during the 2nd part of the motion?

$$a = \frac{\Delta v}{\Delta t} = \frac{5 \frac{m}{s}}{5 s} = 1 \frac{m}{s^2}$$

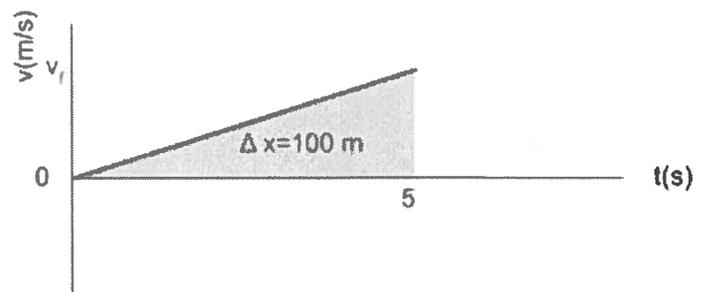


- b. How far did he run, in total?

• Before speeding up:  $\Delta x = \left(5 \frac{m}{s}\right) (8 s) = 40 m$

• While speeding up:  $\Delta x = \left(5 \frac{m}{s}\right) (5 s) + \frac{\left(5 \frac{m}{s^2}\right) (5 s)^2}{2} = 37.5 m$ , so total is  $40 m + 37.5 m = 77.5 m$

7. A mountain goat starts a rock slide and the rocks crash down the mountain 100 m. If the rocks reach



the bottom in 5 s, what was their acceleration?

Method 1: Two steps:

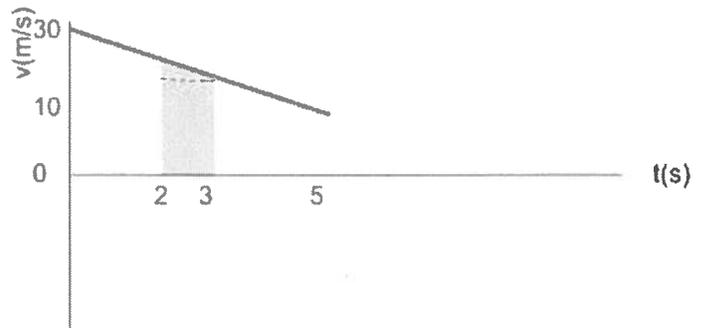
- $\frac{v_f \cdot 5 \text{ s}}{2} = 100 \text{ m}$ , so  $v_f = \frac{2 \cdot 100 \text{ m}}{5 \text{ s}} = 40 \frac{\text{m}}{\text{s}}$
- $a = \frac{\Delta v}{\Delta t} = \frac{40 \frac{\text{m}}{\text{s}}}{5 \text{ s}} = 8 \frac{\text{m}}{\text{s}^2}$

Method 2: time-dependent equation:

$$100 \text{ m} = 0 + \frac{a}{2}(5 \text{ s})^2 = a \cdot 12.5 \text{ s}^2, \text{ so } a = \frac{100 \text{ m}}{12.5 \text{ s}^2} = 8 \frac{\text{m}}{\text{s}^2}$$

8. A car whose initial speed is 30 m/s slows uniformly to 10 m/s in 5 s.

a. Determine the acceleration of the car.



$$a = \frac{\Delta v}{\Delta t} = \frac{-20 \frac{\text{m}}{\text{s}}}{5 \text{ s}} = -4 \frac{\text{m}}{\text{s}^2}$$

b. Determine the distance it travels in from  $t = 2\text{s}$  to  $t = 3\text{s}$ . There are several pathways to an answer here. All of them will involve multiple steps. I'll give a couple.

Method 1-Area under the graph. To find the area, I'm going to divide it into a triangle and a rectangle, as shown on the graph.

- The area of the rectangle is 1 second \* the velocity at 3 s. But

$$v = v_i + a\Delta t = 30 \frac{\text{m}}{\text{s}} + \left(-4 \frac{\text{m}}{\text{s}^2}\right) \cdot 3 \text{ s} = 18 \frac{\text{m}}{\text{s}}$$

$$v = v_i + a\Delta t = 30 \frac{m}{s} + \left(-4 \frac{m}{s}\right) \cdot 3 s = 18 \frac{m}{s}, \text{ so}$$

$$\Delta x_{rect} = \left(18 \frac{m}{s}\right) \cdot (1 s) = 18 m$$

- The area of the triangle is half of 1 second \* the difference between the velocities

$$v = v_i + a\Delta t = 30 \frac{m}{s} + \left(-4 \frac{m}{s}\right) \cdot 2 s = 22 \frac{m}{s}$$

at 2 s and 3 s.

$$\Delta x_{tri} = \frac{\left(22 \frac{m}{s} - 18 \frac{m}{s}\right) (1 s)}{2} = 2 m$$

- The total displacement is the sum of these two displacements, so

$$\Delta x = 18 m + 2 m = 20 m$$

Method 2-Use time-dependent equation twice:

- At 2 seconds:

$$\Delta x = v_i \Delta t + \left(\frac{a}{2}\right) \cdot \Delta t^2 = \left(30 \frac{m}{s}\right) (2 s) + \left(\frac{-4 \frac{m}{s}}{2}\right) (2 s)^2 = 52 m$$

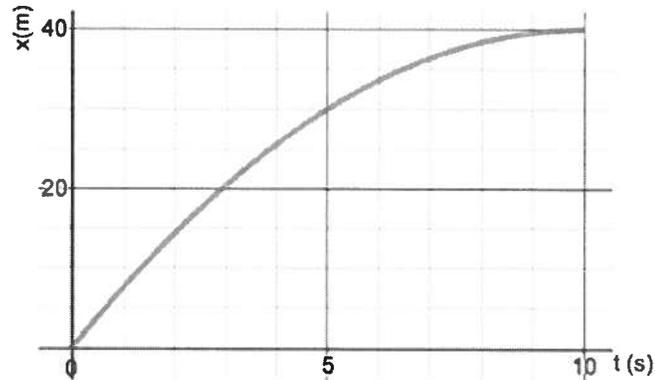
- At 3 seconds:

$$\Delta x = v_i \Delta t + \left(\frac{a}{2}\right) \cdot \Delta t^2 = \left(30 \frac{m}{s}\right) (3 s) + \left(\frac{-4 \frac{m}{s}}{2}\right) (3 s)^2 = 72 m$$

- So during that second:  $\Delta x = 72 m - 52 m = 20 m$

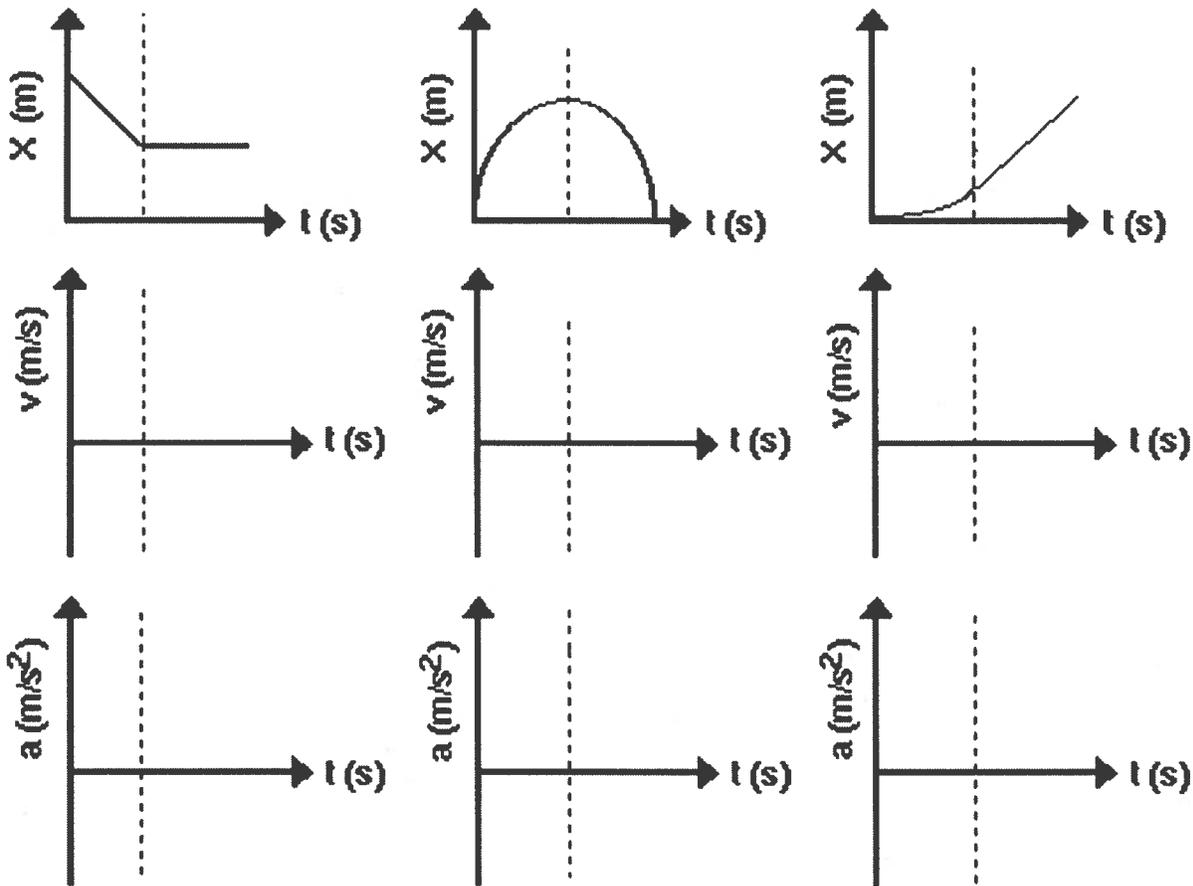
# CAPM Unit Review

Use the graph below to answer questions #1-4 that follow:

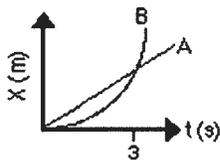


1. Give a written description to describe the motion of this object.
2. Draw the motion map for the object. Include velocity and acceleration vectors.
3. Explain how you could determine the instantaneous velocity of the object at  $t = 2$  s.
4. If an object moving at an initial velocity of 50 m/s comes to a stop in 20 s, what is its acceleration?
5. A Pontiac Trans-Am, initially at rest, accelerates at a constant rate of  $4.0 \text{ m/s}^2$  for 6 s. How fast will the car be traveling at  $t = 6$  s?
6. A tailback initially running at a velocity of 5.0 m/s becomes very tired and slows down at a uniform rate of  $0.25 \text{ m/s}^2$ . How fast will he be running after going an additional 10 meters?

7. For each of the position vs time graphs shown below, draw the corresponding  $v$  vs  $t$ ,  $a$  vs  $t$ , and motion map.



8. Using the graph below, compare the kinematic behavior of the two objects.



**Comparison:**

is  $A > B$ ,  $A < B$ , or  $A = B$ ,

**How do you know?**

- a. Displacement at 3 s
- b. **Average** velocity from 0 - 3 s
- c. **Instantaneous** velocity at 3 s

## UNIT II: Review

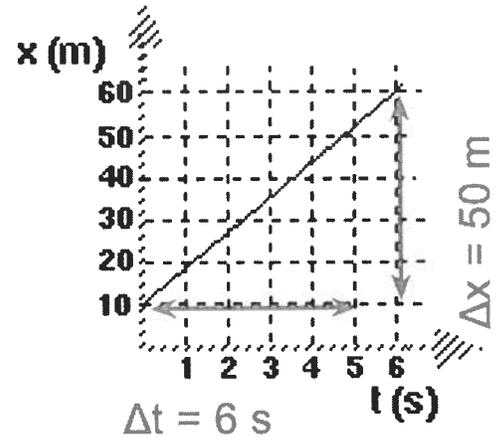
1. Consider the position vs time graph at right.

a. Determine the average velocity of the object.

The slope of the x vs. t graph is the velocity, so:  $v = \frac{\Delta x}{\Delta t} =$   
 $\frac{50 \text{ m}}{6 \text{ s}} = 8.3 \text{ m/s}$

b. Write a mathematical equation to describe the motion of the object.

$$x = (8.3 \text{ m/s})t + 10 \text{ m}$$



2. Shown at right is a velocity vs time graph for an object.

a. Describe the motion of the object.

The object moves forward for 2 seconds at a constant speed of 4 m/s, then moves backwards for 1 second at a constant speed of 3 m/s, then stands still for 2 seconds.

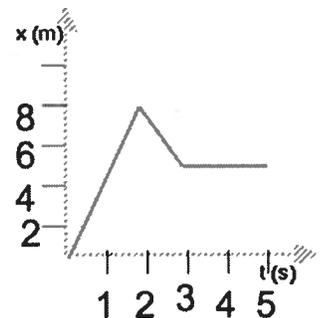
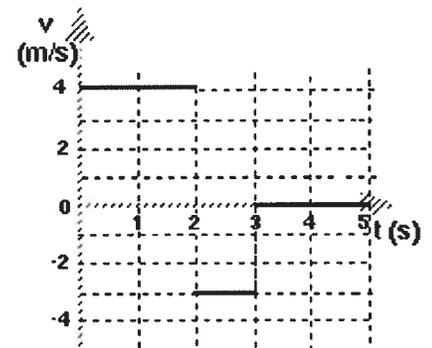
b. Draw the corresponding position vs time graph. Number the x - axis. My graph assumes that the object started at  $x = 0$ .

c. How far did the object travel in the interval  $t = 1\text{s}$  to  $t = 2\text{s}$ ?

At  $t = 1\text{s}$  its position was 4 m and at  $t = 2\text{s}$ , its position was 8m, so it travelled 4 m.

d. What is the total displacement? Explain how you got the answer.

At  $t = 0\text{s}$ , its position was 0m and at  $t = 5\text{s}$ , its position was 5m, so the total displacement was 5m.



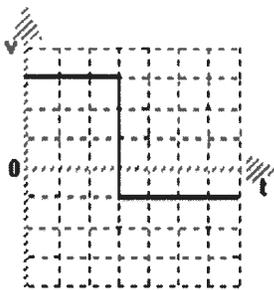
3. Johnny drives to Wisconsin (1920 miles) in 32 hours. He returns home by the same route in the same amount of time.

a. Determine his average speed. Speed doesn't depend on direction (sign, in vector terms), only on distance (magnitude, in vector terms). So we could calculate speed as  $v$  by  $v = \frac{\sum|\Delta x|}{\Delta t}$ . In this case, this is  $v = \frac{1920 \text{ miles} + 1920 \text{ miles}}{32 \text{ hours} + 32 \text{ hours}} = \frac{3840 \text{ miles}}{64 \text{ hours}} = 60 \text{ mi/hr}$ .

b. Determine his average velocity. Velocity does depend on direction and is defined as  $v = \frac{\Delta x}{\Delta t}$ . In this case, velocity is  $v = \frac{1920 \text{ mi} + (-1920 \text{ mi})}{32 \text{ hr}} = \frac{0 \text{ mi}}{32 \text{ hr}} = 0 \text{ mi/hr}$ .

c. Compare these two values and explain any differences. The average velocity is zero because the two movements cancel each other out to result in 0 overall displacement.

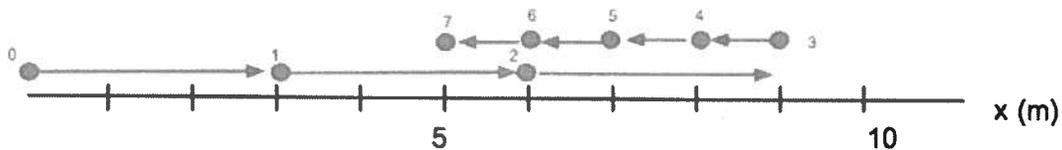
4. Consider the v vs t graph below.



a. Describe the behavior of the object depicted in the graph.

Assuming the velocity unit is m/s and the time unit is s, it moves forward at 3 m/s for 3 seconds, then turns around and moves backwards at 1 m/s for the next 4 seconds.

b. Draw a motion map that represents the behavior of the object.



5. A race car travels at a speed of 95 m/s. How far does it travel in 12.5 s? Use the appropriate mathematical expression and show how units cancel. (Keep the proper number of sig. figs.) Since  $v = \frac{\Delta x}{\Delta t}$ , it must also be true that  $\Delta x = v\Delta t = \left(\frac{95 \text{ m}}{1 \text{ s}}\right) \left(\frac{12.5 \text{ s}}{1}\right) = 1200 \text{ s}$ . We would keep 2 sig. figs. in our answer, because there are only 2 sig. figs. in 95 m/s.

# VPython and Glowscript

Welcome to the computational side of physics!

## Lesson 3: Acceleration and Simulation Accuracy

**Expected time: 1 hour**

**Objectives: In this tutorial you will;**

- Learn how to accelerate objects with vpython
- Evaluate the effectiveness of computational modeling compared to analytical solutions

### Table of Contents

Lesson 3: Acceleration and Simulation Accuracy

Adding acceleration

Your Task

Final Product

### Adding acceleration

We have learned in class that motion with constant acceleration can be modeled using the equation  $x = x_0 + v_0\Delta t + \frac{1}{2}a\Delta t^2$ . This effectively changes the position of an object moving in constant velocity ( $x = x_0 + v\Delta t$ ) using the acceleration term ( $\frac{1}{2}a\Delta t^2$ ). However, this is not the only, nor the best, method of accounting for velocity changing, particularly because it only works for an object accelerating constantly.

Instead we will update the velocity itself.

This is a powerful idea, as we can do this for any situation with no matter the acceleration.

Recall from lesson 2 that the method for updating position, `car.pos=car.pos+car.v*dt`, was important; now we will see why.

Let's take a car accelerating at a rate of 2 m/s/s as an example. If it starts at rest and accelerates for 5 seconds, the final position will be  $x = (0\text{ m}) + (0\text{ m/s}) \cdot (5\text{ s}) + \frac{1}{2}(2\text{ m/s/s}) \cdot (5\text{ s})^2 = 25\text{ meters}$ . Instead, what if we update the velocity every second and use that to find the new position? After the first second the car will be traveling at 2 m/s, so then it travels 2 meters during that second. After the next, it is moving 4 m/s, so it travels 4 meters. This is summarized in the table below .

Time (s)	Velocity (m/s)	Approximate Displacement (m) in previous second ( $v \cdot dt$ )	Final position (m)
0	0	0	0
1	2	2	2
2	4	4	6
3	6	6	12
4	8	8	20
5	10	10	30

A couple of things are important here. First, the final position takes the previous position and adds the next displacement, which is the same as  $\text{car.pos} = \text{car.pos} + \text{car.v} \cdot dt$ . So to find the position at time 2 s, take the position at 1 s, which is 2 m, find the displacement from 1 to 2 seconds as the velocity times the time, so  $(4\text{ m/s}) \cdot (1\text{ s}) = 4\text{ m}$ . That gets added to the previous position to give the new position of 6 meters.

Wait a minute. Didn't we find that the final position should be 25 meters? The problem is that from zero to 1 second, for example, the car wasn't actually moving at 2 m/s the whole time; it's average velocity was actually 1 m/s. Here's where the computational modeling comes in; making the time interval smaller increases the accuracy of the simulated position. Changing the time interval from one second to a half second yields a final position of 27.5 m, and going to 0.1 s yields 25.5 meters, as shown in [this spreadsheet](#).

So how small of a change in time is small enough? That's what we're going to work on now. The problem is that if we make dt too small, the program will bog down and run very slowly.

## Your Task

Using a car starting at rest with an acceleration of 2 m/s/s, modify your loop to first update the velocity (`car.v`) before updating the position. It will be helpful to know that vPython includes a definition for acceleration (a vector quantity). The syntax looks like:

```
car.a=vec(.225,0,0) #assigns an acceleration in the (x,y,z) directions
```

Once you get it working, print a calculation of the *actual* final position as well as the final position according to the simulation. Experiment with `dt`, finding a value that seems to accurately predict the final position without being too small. When you get something that you feel is a good compromise between accuracy and laginess, make a screencast that shows your code and the code running. This screencast should be less than two minutes.

Once you complete the main task, try again with an acceleration of 10 m/s/s instead; does this change anything? Another thing you could try is making the acceleration negative and changing the initial velocity so it isn't zero; try large initial velocities (these changes will mess with your graphs, but the printed values should work if you do them correctly).

## Final Product

You will be turning in your screencast. This, plus another short independent coding test will be used to demonstrate proficiency in standard CMPH.1.

## Introduction to the Balanced Force Particle Model

Up to this point in physics, the models we have been developing have mostly been what could be called “descriptive” models. In other words the models describe **what** is happening in constant velocity and constant acceleration motion, but they do not provide an explanation for **why** the object would be in either kind of motion.

In this unit, we will be developing an “predictive” model. This is a different type of model because its purpose is **not to describe** motion but to predict which type of motion the object should be under.

### Force

The main tool for this prediction is something called force. Force is one of those words that is used in everyday language to mean a lot of different things, but to a physicist it has a very specific definition. We'll actually have to use the idea of force before we have the ideas needed for that specific definition, so for now, we'll use this working definition: **A force is a push or a pull by one object on another.** Forces are vector quantities, like velocity and acceleration, because they have not only a size, but also a direction.

One way to categorize types of forces is by whether they act because of a contact or without contact. Most forces arise from contact with other objects. There are three specific types of forces that exist without contact:

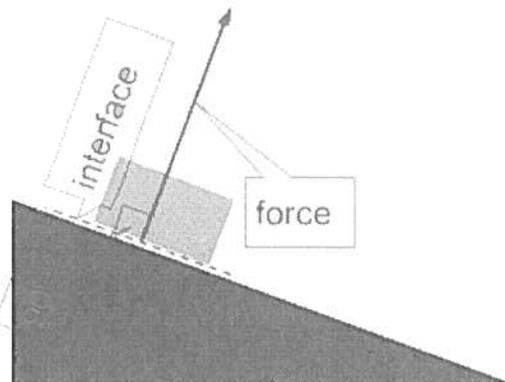
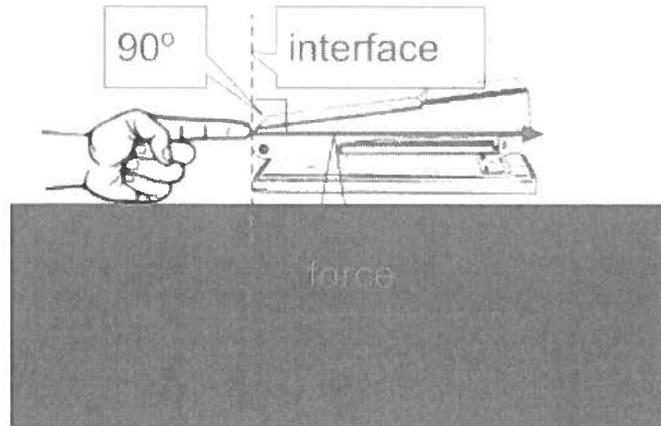
- The force due to gravity: All objects pull on all other objects because of gravity, even if they aren't in contact. We only really notice the pull when the objects are really big...like planets. This is the one we will see in this unit. We'll give it the label  $F_g$ . The direction of  $F_g$  is always between the centers of the two objects pulling each other.
- Magnetic force: If you hold the north and south poles of two magnets apart from each other, you can feel a pull between them. If you press the north poles of two different magnets toward each other, you can feel them push away from each other. Both these things happen even though the magnets are not in contact.
- Electrostatic force: A long time ago, people discovered that if you rubbed a piece of glass with some silk, it would attract tiny things like bits of paper or hair, even without contact. They also saw that if you rubbed some amber with fur, the same thing would happen (this also works with plastic and fur). This attractive force is called an electrostatic force. This type of force is extremely important when the objects are the very tiny things called atoms that the whole universe is made of and the study of how it explains nearly everything that happens with matter is called chemistry.

There are three specific types of contact forces:

- Tension force: This is any pulling force. We give this type of force the label  $F_T$ . A pulling force always acts in the direction from the object being pulled toward the object doing the pulling.

- Normal force: This is any pushing force. We give this type of force the label  $F_N$ . The definition of “normal” in this context is “perpendicular”. If you look at where the object being pushed and the object doing the pushing meet (called the “interface”), the direction of the normal force is perpendicular to that interface. Here are a couple of examples:

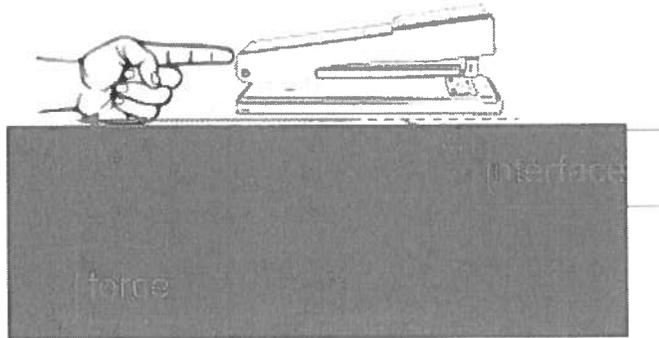
- A stapler being pushed across a desk by a finger:



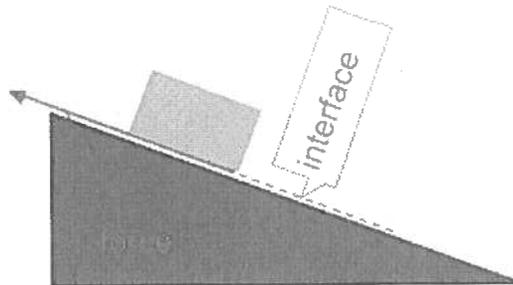
- A box being pushed up by a ramp:

- Frictional force: This is a force that is so common, we sometimes don't think of it as a force, but rather a property of motion. This causes problems for beginning physics students. When an object is moving, or is being pushed or pulled in a way that could cause moving, friction is a force by a contacting surface that resists that movement. We give frictional force the label  $F_f$ . The direction of frictional force is always parallel to the interface between the object and the surface and opposite motion whether the motion is happening or not. The examples from above are repeated here to show the direction of frictional force.

- A stapler being pushed across a desk by a finger:



- A box sliding down a ramp:



### Force and Motion

What we learn by analyzing motion very carefully are two facts that never seem to have an exception. This is such an important model that scientists have called it a law. Isaac Newton was the first scientist to formally point this out, so this is called **Newton's First Law of Motion (or NL1, for short)**:

- **When all the forces on an object balance each other out, the object moves at a constant velocity, which includes a velocity of zero (not moving).**
- **When the forces on an object are not balanced, the object is accelerating.**

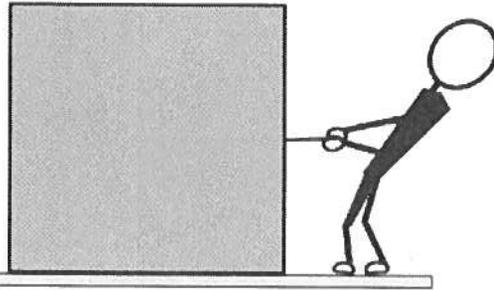
### Free-Body Diagrams (FBDs)

In order to help describe the forces on an object to predict the type of motion or to find the size of unknown forces, scientists and engineers use a tool called a "**free-body diagram**" to model the forces on an object. It's a powerful sketch that has the following features:

- The object whose motion we are investigating (called the "system") is modeled as a single dot.
- All forces on the object are modeled as arrows:
  - The direction of the arrow is the direction of the force.
  - The size of the arrow is the size of the push or pull.
  - Labels ( $F_g$ ,  $F_T$ ,  $F_N$  and  $F_f$ ) tell what kind of force it is. In addition, we are going to add to our labels something called a "by-on". It's just a set of parentheses that includes the object the force is caused "by" then a comma and the object the force acts "on". For example, if a string pulls on a box, we would label that vector

$$F_{g(\text{string, box})}$$

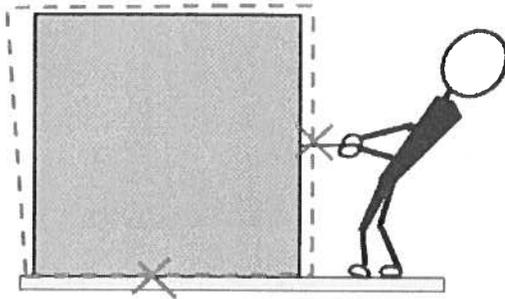
Here's how to draw a successful FBD, with an example. Say a man is pulling a box at constant



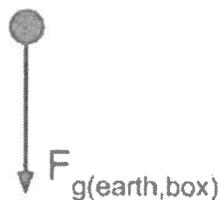
speed across a floor:  
box.

Our job is to draw a FBD of the

1. Create a system schema by drawing a circle around the object for which you are making the FBD, then marking any places where the system contact other objects:

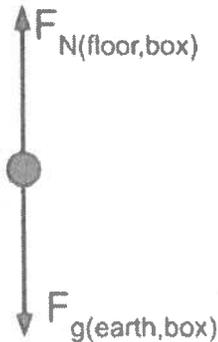


2. Draw a dot that represents the system: 
3. Add an arrow that represents the force due to gravity and label it. In all cases on earth, this will be present and it always acts toward the center of the earth, so straight down:

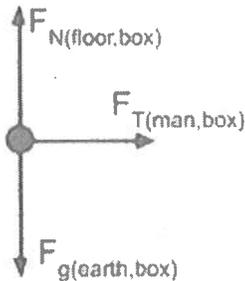


4. Add a force for each of the contacts you marked. As you do, think about NL1 and the state of motion of the object.

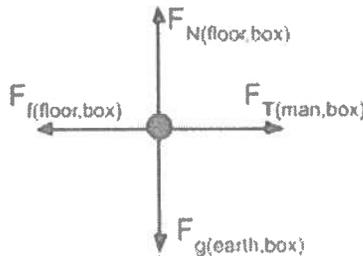
- a. Since this box is moving at constant speed, the forces need to be balanced, so my interface with the floor must provide a normal force the same size as  $F_g$ :



- b. The man provides a pulling force marked on the right of my system schema:



- c. But NL1 says I have to balance this force. I can't provide any forces that don't come from contacts, so this balancing force must be a frictional force provided by



the floor:

### Using FBDs and NL1 to find missing forces:

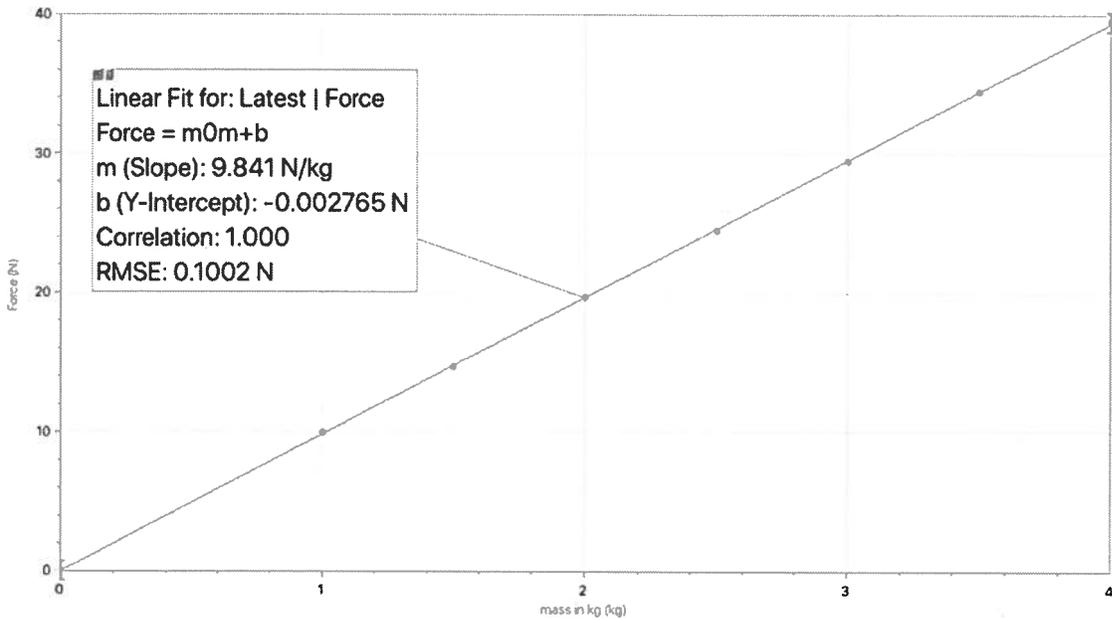
Because all forces have to be balanced, we can use a FBD as a problem-solving tool for situations in which the object is moving at constant velocity (which includes not moving). For example, say the man has to pull the box with a force of 30 Newtons (N) (yeah, Newton is such a big deal they named the force unit after him). This means that the force of friction has to also be 30 N also.

### Weight and Mass:

We measure how much stuff is present in an object using mass. The standard unit of mass is the kilogram (kg). In the US system, this represents a bit over 2 pounds. And if you ask

someone from Canada or Europe how much they weigh, they will tell you a number of kilograms. But kilograms isn't a force and weight is a force--it's actually what  $F_g$  is. Your mass is the same on earth and the moon, but your weight is much less on the moon, because the gravitational force is smaller there.

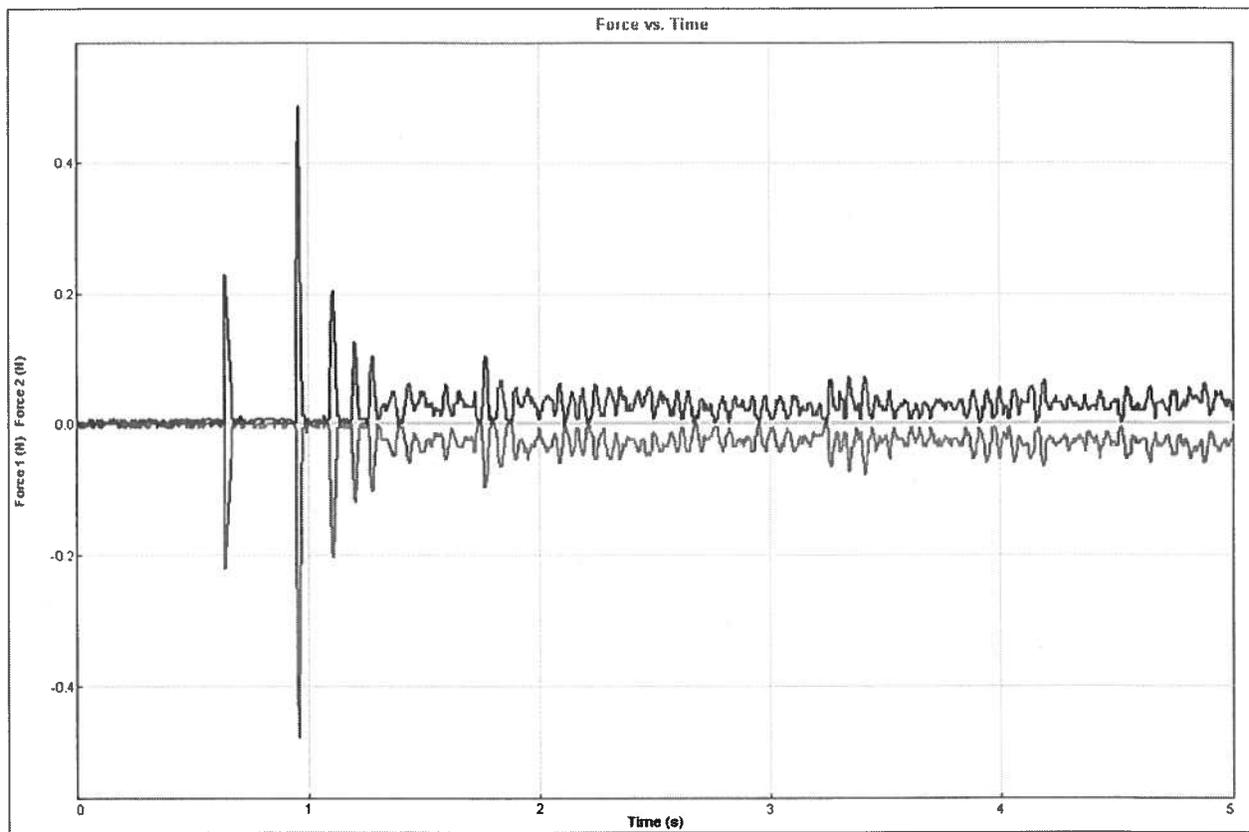
If you take a lot of objects of different masses and use a force sensor to find their weights, you find something like this graph of mine from a few years ago:



This shows that the weight of an object in Newtons ( $F_g$ ) is about 10 times the mass of the object in kilograms. (9.8 if you want to be more precise). This is known as earth's gravitational field strength and you can use it to find the weight of an object if you know its mass.

### Newton's 3rd Law (NL3)

When one object exerts a force on another, the second object exerts a force back that is equal in size, but opposite in direction. This finding is known as Newton's Third Law (NL3 for short). This is not a force-balance thing, like with NL1, because it turns out to be true no matter what the state of motion was. Here's one example of this law in action. I had two students use force sensors to pull against each other in a tug-of-war. Here's the result:



The student whose force sensor shows as the blue graph won the tug-of-war easily. But as you can see, the forces match almost perfectly. This is counter-intuitive, because it seems like the winner should have exerted more force. But the effect of the force is not the same as the actual force.

# Study Guide for Introduction to the Balanced Force Particle Model

What is the difference between the models we had before and the model we will use in this unit?

## Force

What is a good working definition for force?

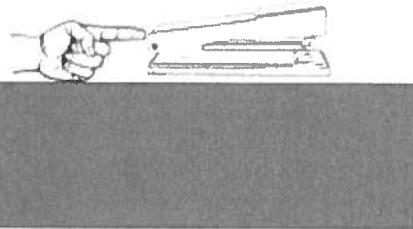
What are the three non contact forces and a brief description of each:

- -
  
- -
  
- -

What are the three non contact forces and a brief description of each:

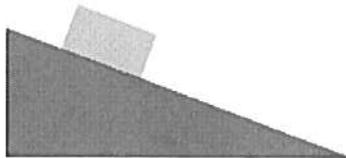
- -
  
- -

- Draw the direction of the normal force for a stapler being pushed across a desk



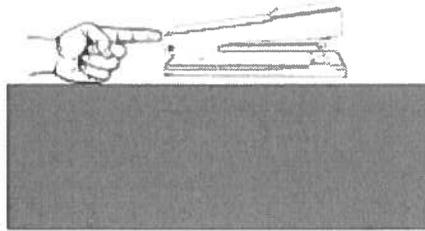
by a finger:

- Draw the direction of the normal force for a box being pushed up by a ramp:



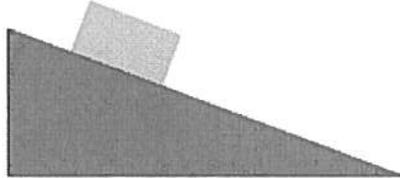
- -

- Draw the direction of the frictional force on a stapler being pushed across a desk



by a finger:

- Draw the direction of the frictional force on a box sliding down a ramp:



### Force and Motion

What are the two statements that make up **Newton's First Law of Motion (NL1)**:

- -
- -

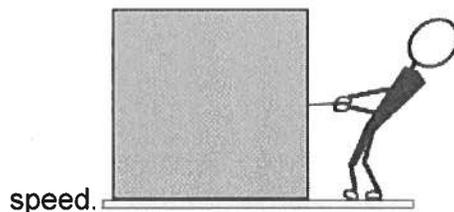
### Free-Body Diagrams (FBDs)

In a FBD how do we model ...

- the object whose motion we are investigating?
- the forces on the object?

What is a "by-on"?

- Draw a system schema for this situation where a man is pulling a box at a constant



- Draw and label the forces on the box to make a FBD:

### Using FBDs and NL1 to find missing forces:

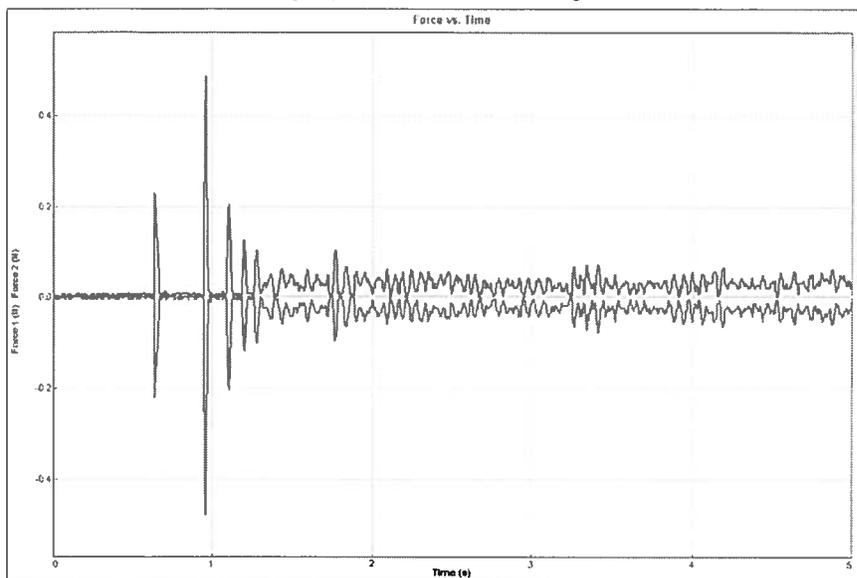
How does NL1 and a FBD allow us to find the force of friction if the man pulls with a force of 30 N?

### Weight and Mass:

- What is the difference between weight and mass?
- How can you find the weight of an object on earth if you know its mass?

### Newton's 3rd Law (NL3)

- What is NL3?
- How does this graph of forces in a tug-of-war illustrate NL3?

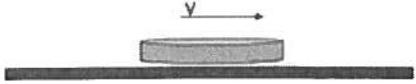


The student whose force sensor shows as the blue graph won the tug-of-war easily.

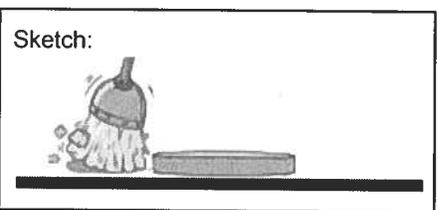
## Introduction to Forces Model-Building Project

In each of the following exercises, the "puck" is a hovercraft puck that slides on a frictionless cushion of air when it is turned on. When it is turned off, there is friction.

Description: The puck is on, sitting at rest on a level floor, and not being pushed or pulled horizontally.	Sketch: 
Force Diagram:	$\sum F =$ <hr style="width: 80%; margin-left: 0;"/> The puck will...
Motion Map: 	

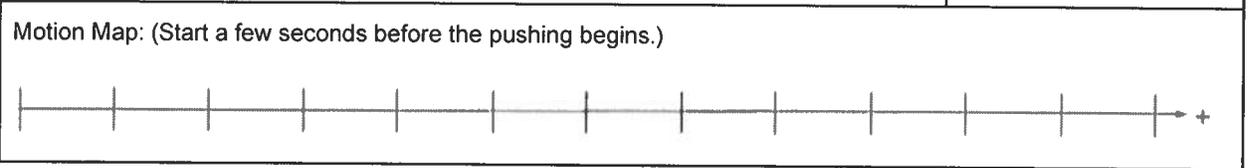
Description: The puck is on, moving to the right on a level floor, and not being pushed or pulled horizontally.	Sketch: 
Force Diagram:	$\sum F =$ <hr style="width: 80%; margin-left: 0;"/> The puck will...
Motion Map: 	

Description: The puck is on, starts out sitting still on a level floor, and then is pushed with constant force to the right by a broom. Draw your FBD after the broom begins to push.

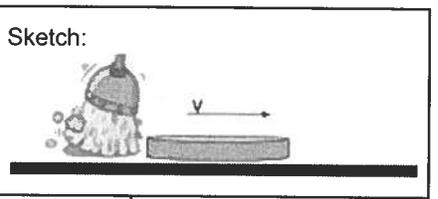


Force Diagram:

$\sum F =$  \_\_\_\_\_  
 The puck will...

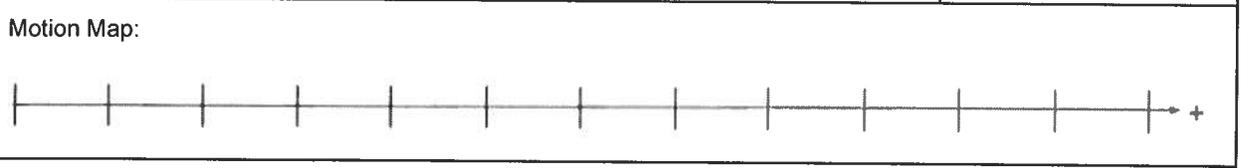


Description: The puck is on, moving to the right on a level floor, and then is pushed with constant force to the right by a broom. Draw your FBD after the broom begins to push.



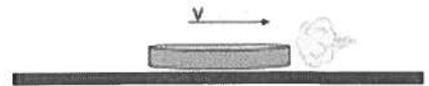
Force Diagram:

$\sum F =$  \_\_\_\_\_  
 The puck will...



Description: The puck is on, moving to the right on a level floor, and is being pushed constantly to the left by a breeze from a fan.

Sketch:



Force Diagram:

$$\sum F =$$

The puck will...

Motion Map:



Description: The puck is off, sitting at rest on a level floor, and not being pushed or pulled horizontally.

Sketch:



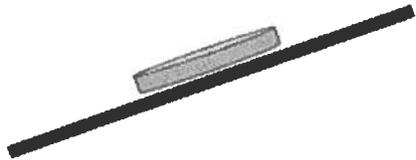
Force Diagram:

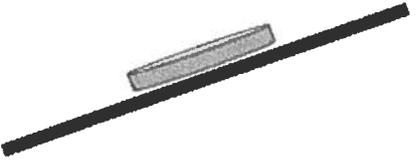
$$\sum F =$$

The puck will...

Motion Map:

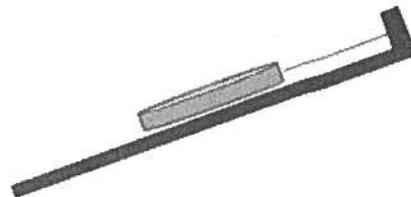


<p>Description: The puck is turned off and placed on a ramp. It doesn't move.</p>	<p>Sketch:</p> 
<p>Force Diagram:</p>	<p><math>\sum F =</math> _____</p> <p>The puck will...</p>
<p>Motion Map ("+" is up the ramp):</p> 	

<p>Description: The puck is turned on, placed on a ramp and released from rest.</p>	<p>Sketch:</p> 
<p>Force Diagram:</p>	<p><math>\sum F =</math> _____</p> <p>The puck will...</p>
<p>Motion Map ("+" is up the ramp):</p> 	

Description: The puck is on, sitting on a ramp, tied to a string which is anchored to the top of the ramp.

Sketch:



Force Diagram:

$$\sum F =$$

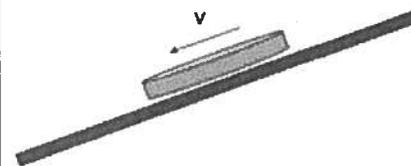
The puck will...

Motion Map ("+" is up the ramp):



Description: The puck is on, sliding down the ramp when the hovercraft motor's battery suddenly dies.

Sketch:



Force Diagram (Draw this for the instant right after the battery dies):

$$\sum F =$$

The puck will...

Motion Map ("+" is up the ramp): (Start this a few seconds before the battery dies.)

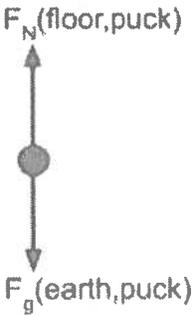


Conclusions:

- Whenever an object that is at rest experiences a net force of zero, it will...
- Whenever an object that is at rest experiences a net force in a particular direction it will...
- Whenever a moving object experiences a net force of zero, it will...
- Whenever an object moving in a particular direction experiences a net force in the direction it is moving, it will...
- Whenever an object moving in a particular direction experiences a net force in the direction opposite to the direction it is moving, it will...

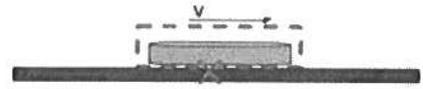
## Introduction to Forces Model-Building Project

In each of the following exercises, the “puck” is a hovercraft puck that slides on a frictionless cushion of air when it is turned on.

Description: The puck is on, sitting at rest on a level floor, and not being pushed or pulled horizontally.	Sketch: 
Force Diagram: system schema shows only one contact...the floor 	$\sum F = 0$ <p>The puck will...remain still on the floor b/c balanced forces mean no accel.</p>
Motion Map: 	

Description: The puck is on, moving to the right on a level floor, and not being pushed or pulled horizontally.

Sketch:



Force Diagram: system schema shows only one contact...the floor

$F_N(\text{floor,puck})$



$F_g(\text{earth,puck})$



$$\sum F = 0$$

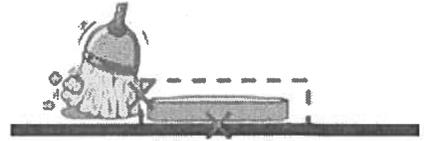
The puck will...continue moving to the right b/c balanced forces mean no accel.

Motion Map:

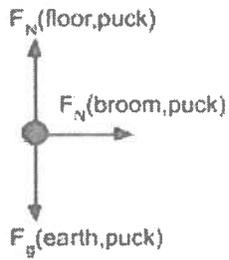


Description: The puck is on, starts out sitting still on a level floor, and then is pushed with constant force to the right by a broom. Draw your FBD after the broom begins to push.

Sketch:



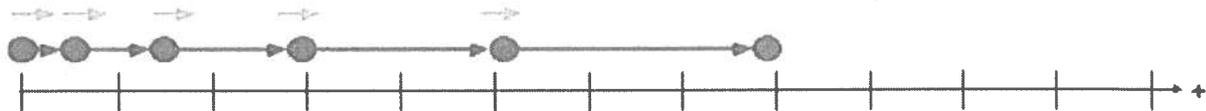
Force Diagram: system schema shows two contacts...the floor and the broom.



$$\sum F = F_{N(\text{broom,puck})}$$

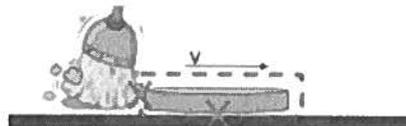
The puck will...start moving and speed up to the right, b/c the unbalanced force will cause acceleration in that direction

Motion Map:

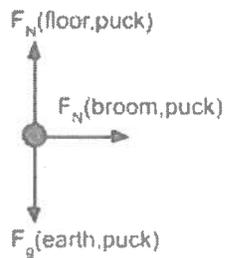


Description: The puck is on, moving to the right on a level floor, and then is pushed with constant force to the right by a broom. Draw your FBD after the broom begins to push.

Sketch:



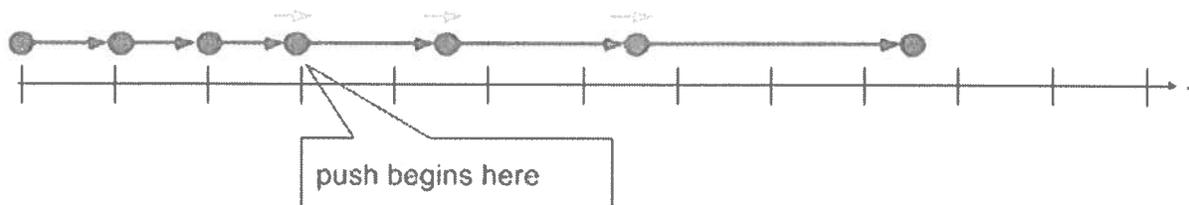
Force Diagram: system schema shows two contacts...the floor and the broom.



$$\sum F = F_{N(\text{broom,puck})}$$

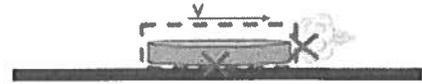
The puck will...speed up to the right, b/c the unbalanced force will cause acceleration in that direction and it's already going that direction.

Motion Map: (Start a few seconds before the pushing begins.)

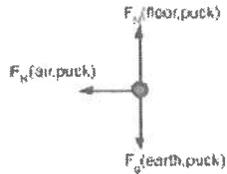


Description: The puck is on, moving to the right on a level floor, and is being pushed constantly to the left by a breeze from a fan.

Sketch:



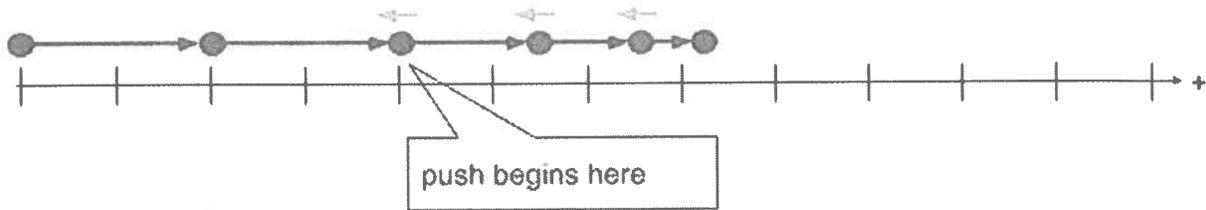
Force Diagram: system schema shows two contacts...the floor and the air.



$$\sum F = F_{N(\text{air,puck})}$$

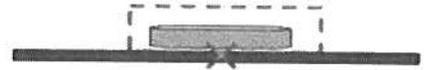
The puck will...slow down, b/c the unbalanced force will cause acceleration to the left in that direction and it's moving the opposite direction.

Motion Map:



Description: The puck is off, sitting at rest on a level floor, and not being pushed or pulled horizontally.

Sketch:



Force Diagram: system schema shows only one contact...the floor

$F_N(\text{floor,puck})$



$F_g(\text{earth,puck})$

$$\sum F = 0$$

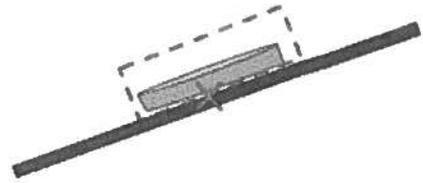
The puck will...remain still on the floor b/c balanced forces mean no accel.

Motion Map:

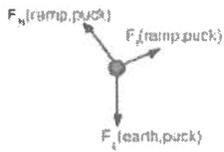


Description: The puck is turned off and placed on a ramp. It doesn't move.

Sketch:



Force Diagram: system schema shows only one contact...the ramp. Normal forces are perpendicular to the interface and frictional forces are parallel to the interface, and this surface will have a frictional force b/c the air is turned off.



$$\sum F = 0$$

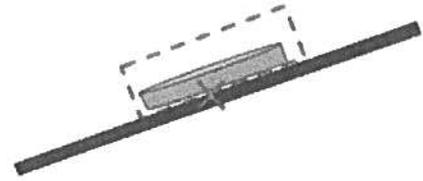
The puck will...Remain stationary on the ramp b/c the balanced forces will cause no acceleration. If you assumed that the  $F_f$  doesn't balance gravity (also a possible right answer) then the puck accelerates down the ramp.

Motion Map ("+" is up the ramp):

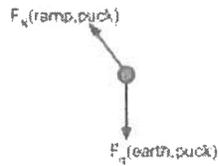


Description: The puck is turned on, placed on a ramp and released from rest.

Sketch:



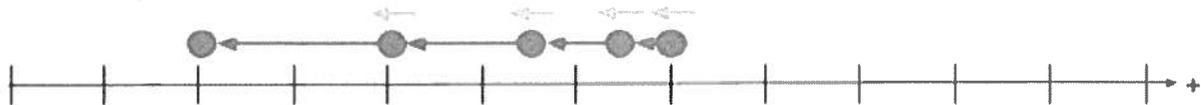
**Force Diagram:** system schema shows only one contact...the ramp. Normal forces are perpendicular to the interface and frictional forces are parallel to the interface, but this surface will have no frictional force b/c the puck is turned on.



$\sum F =$  the part of  $F_g$  that is not balanced by  $F_N$

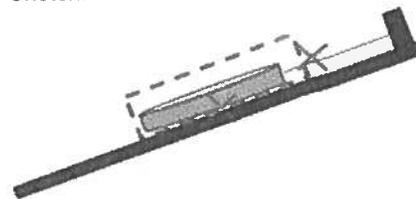
The puck will...speed up down the ramp because that is the direction of the net force

Motion Map ("+" is up the ramp):

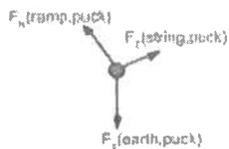


Description: The puck is on, sitting on a ramp, tied to a string which is anchored to the top of the ramp.

Sketch:



**Force Diagram:** system schema shows two contacts...the ramp and the string. Normal forces are perpendicular to the interface and frictional forces are parallel to the interface, but this surface will have no frictional force b/c the puck is turned on.



$$\sum F = 0$$

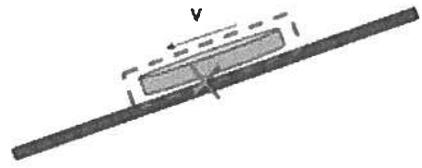
The puck will...Remain stationary on the ramp b/c the balanced forces will cause no acceleration.

Motion Map ("+" is up the ramp):

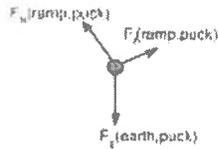


Description: The puck is on, sliding down the ramp when the hovercraft motor's battery suddenly dies.

Sketch:



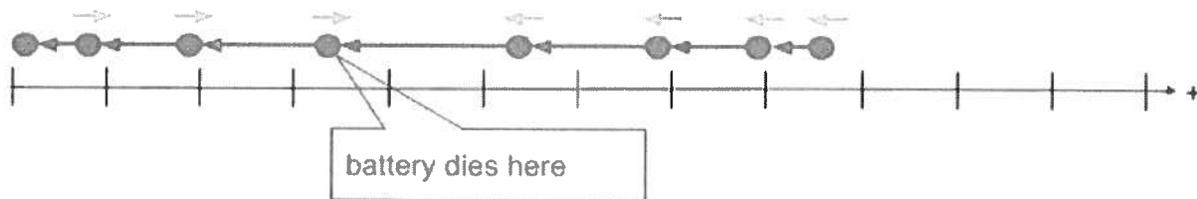
Force Diagram (Draw this for the instant right after the battery dies): system schema shows only one contact...the ramp. Normal forces are perpendicular to the interface and frictional forces are parallel to the interface, and this surface will have a frictional force b/c the air has failed.



$\sum F =$  the difference between the part of  $F_g$  that acts down the ramp and the  $F_f$

The puck will... Slow down b/c I assume  $F_f > F_g$  down the ramp. If you assumed that  $F_g > F_f$  (also a possible right answer) then the puck accelerates down the ramp.

Motion Map ("+" is up the ramp): (Start this a few seconds before the battery dies.)



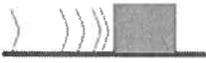
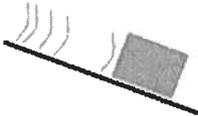
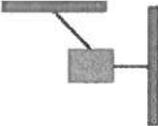
Conclusions:

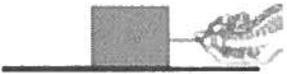
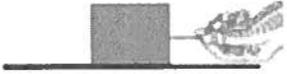
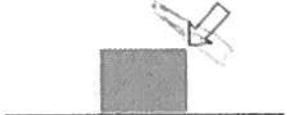
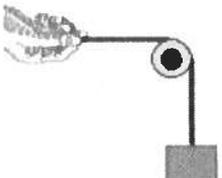
- Whenever an object that is at rest experiences a net force of zero, it will... remain at rest
- Whenever an object that is at rest experiences a net force in a particular direction it will... begin moving in that direction at an increasing speed

- Whenever a moving object experiences a net force of zero, it will...continue moving at its current speed
- Whenever an object moving in a particular direction experiences a net force in the direction it is moving, it will...speed up
- Whenever an object moving in a particular direction experiences a net force in the direction opposite to the direction it is moving, it will...slow down

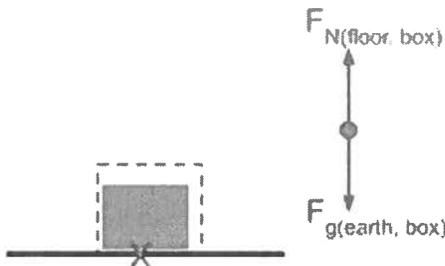
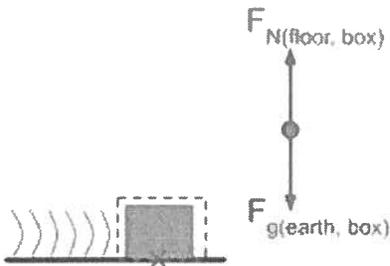
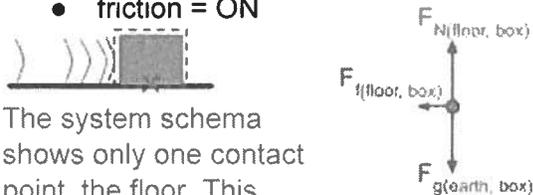
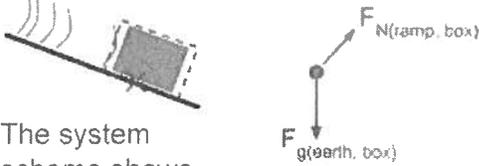
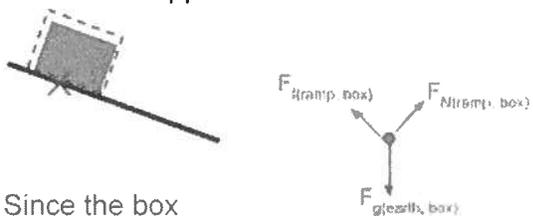
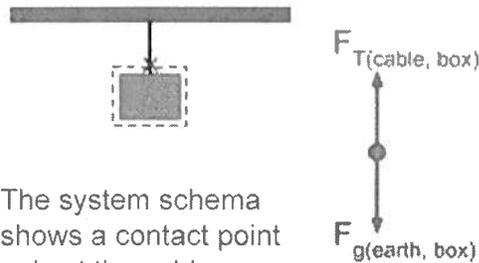
## BFPM Practice and Exploration #1

In each box below, there's a drawing that represents an object in some state of motion. Other information about the conditions on the object is also given. Draw a force diagram (or "free-body diagram", FBD) that models all the forces acting on the object with vectors. Do your best to make the lengths of the vectors represent the relative magnitudes of the forces.

<ul style="list-style-type: none"> <li>• box appears motionless</li> </ul> 	<ul style="list-style-type: none"> <li>• speed = constant</li> </ul> 
<ul style="list-style-type: none"> <li>• velocity <math>\neq 0</math></li> <li>• friction = ON</li> </ul> 	<ul style="list-style-type: none"> <li>• friction = 0</li> </ul> 
<ul style="list-style-type: none"> <li>• box appears motionless</li> </ul> 	<ul style="list-style-type: none"> <li>• box appears motionless</li> <li>• box hangs by a cable</li> </ul> 
<ul style="list-style-type: none"> <li>• box appears motionless</li> <li>• box hangs by cables</li> </ul> 	<ul style="list-style-type: none"> <li>• box appears motionless</li> <li>• box attached by cables</li> </ul> 
<ul style="list-style-type: none"> <li>• box appears motionless</li> </ul> 	<ul style="list-style-type: none"> <li>• box appears motionless</li> </ul> 

<ul style="list-style-type: none"> <li>• the string is parallel to the surface</li> <li>• friction = OFF</li> </ul> 	<ul style="list-style-type: none"> <li>• the string is parallel to the surface</li> <li>• speed is constant</li> </ul> 
<ul style="list-style-type: none"> <li>• speed is constant</li> </ul> 	<ul style="list-style-type: none"> <li>• friction = OFF</li> </ul> 
<ul style="list-style-type: none"> <li>• box is raised at constant speed</li> <li>• draw FBD for box</li> </ul> 	<ul style="list-style-type: none"> <li>• speed is constant</li> </ul> 
<ul style="list-style-type: none"> <li>• fly ball is rising</li> <li>• ignore air resistance</li> </ul> 	<ul style="list-style-type: none"> <li>• flyball is at the top of its arc</li> <li>• ignore air resistance</li> </ul> 

## BFPM Practice and Exploration #1 Solutions

<ul style="list-style-type: none"> <li>● box appears motionless</li> </ul>  <p>"Motionless" is an example of constant velocity. Therefore the forces on the box must be balanced. The system schema shows only one contact point, the floor. This exerts a normal force the balance is the gravitational force.</p>	<ul style="list-style-type: none"> <li>● speed = constant</li> </ul>  <p>Because the box is at constant velocity, the forces on the box must be balanced. The system schema shows only one contact point, the floor. This exerts a normal force the balance is the gravitational force.</p>
<ul style="list-style-type: none"> <li>● velocity <math>\neq 0</math></li> <li>● friction = ON</li> </ul>  <p>The system schema shows only one contact point, the floor. This exerts both a normal force, which balances the gravitational force, and a force of friction. Since the box is moving and the frictional force is unbalanced, the box experiences acceleration that slows it down.</p>	<ul style="list-style-type: none"> <li>● friction = 0</li> </ul>  <p>The system schema shows only one contact point, the ramp. This exerts a normal force. Since there is no friction and no other contact point this is the only force besides gravitational acting on the box. Since the forces don't balance, the box will accelerate, speeding up.</p>
<ul style="list-style-type: none"> <li>● box appears motionless</li> </ul>  <p>Since the box appears motionless, that is constant velocity. This means forces must be balanced. There is only one contact point and the ramp exerts the normal force and force of friction which balance each other, as well as balancing the force due to gravity.</p>	<ul style="list-style-type: none"> <li>● box appears motionless</li> <li>● box hangs by a cable</li> </ul>  <p>The system schema shows a contact point only at the cable. Since the box is at constant velocity, the forces must be balanced, so the tension force is equal and opposite to the gravitational force.</p>

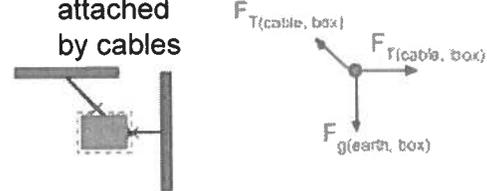
- box appears motionless
- box hangs by cables



The system

schema shows 2 contact points, at the 2 cables. Since the box is at constant velocity, the forces must be balanced. This means the tension forces in the two cables together provides enough upward force to just balance gravitational force. Also each cable's tension force must provide enough horizontal force to just balance the other.

- box appears motionless
- box attached by cables



The system

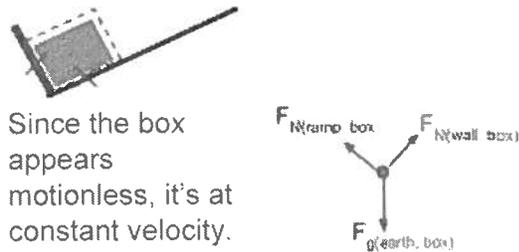
schema shows 2 contact points, at the 2 cables. Since the box is at constant velocity, the forces must be balanced. This means the diagonal cable must provide just enough tension force to balance the tension in the horizontal cable. The diagonal cable must also provide just enough tension force to balance gravitational force.

- box appears motionless



Since the box appears motionless, it's at constant velocity. This means forces must be balanced. There are 2 contact points: The ramp exerts a normal force and the cable exerts a tension force. They must balance each other, as well as balancing the force due to gravity.

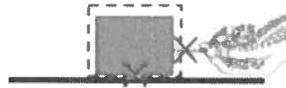
- box appears motionless



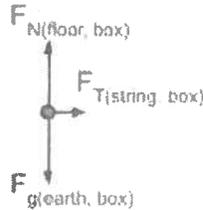
Since the box appears motionless, it's at constant velocity.

This means forces must be balanced. There are 2 contact points: The ramp exerts a normal force and the wall also exerts a normal force. They must balance each other, as well as balancing the force due to gravity.

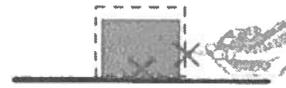
- the string is parallel to the surface
- friction = OFF



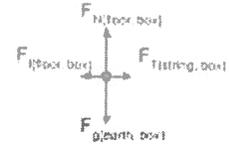
There are only two contact points, the string and the floor. The floor provides normal force, which balances the force of gravity. There is no friction, so the only other force is the tension force from the string, which is unbalanced. This means the box is accelerating.



- the string is parallel to the surface
- speed is constant



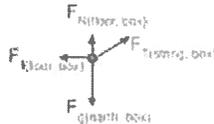
There are only two contact points, the string and the floor. The floor provides normal force, which balances the force of gravity. The box moves at constant velocity, so the tension force from the string, must be balanced by a frictional force.



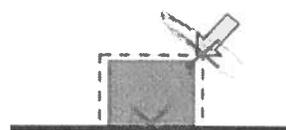
- speed is constant



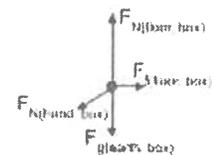
Since the box is at constant velocity, the forces must be balanced. There are two contact points, the floor and the string. Because the string is at an angle, its tension force provides a horizontal balance to the force of friction, and a vertical force that helps balance the force of gravity. Because the force of gravity is balanced only partly by the normal force from the floor, this normal force is smaller than the force of gravity.



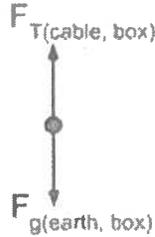
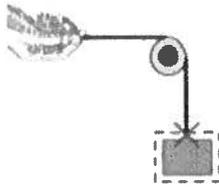
- friction = OFF



Since the box is at constant velocity, the forces must be balanced. There are two contact points, the floor and the hand. Because the hand acts at an angle, its normal force provides a horizontal balance to the force of friction, and a vertical force that helps balance the normal force from the floor. Because the force of gravity and the vertical part of the normal force from the hand are both balanced by the normal force from the floor, this normal force is larger than the force of gravity.

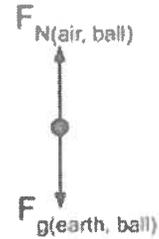


- box is raised at constant speed
- draw FBD for box



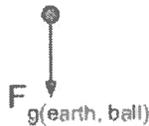
The system schema shows a contact point only at the cable. Since the box is at constant velocity, the forces must be balanced, so the tension force is equal and opposite to the gravitational force.

- speed is constant



The system schema shows a contact point only at the air in front of the moving box. Since the box is at constant velocity, the forces must be balanced, so the normal force from the air is equal and opposite to the gravitational force.

- fly ball is rising
- ignore air resistance



The system schema shows no contact whatsoever, since we are ignoring air resistance. This means the only force acting on the object is the non-contact gravitational force. Since this force is unbalanced, the ball is experiencing acceleration, which slows it down.

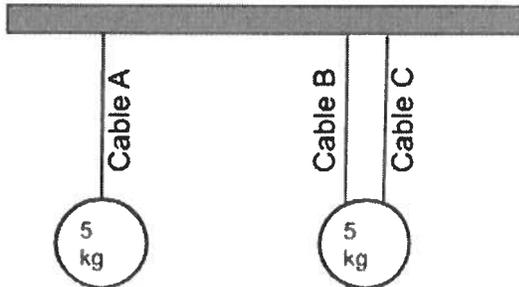
- flyball is at the top of its arc
- ignore air resistance



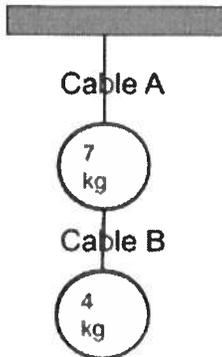
The system schema shows no contact whatsoever, since we are ignoring air resistance. This means the only force acting on the object is the non-contact gravitational force. Since this force is unbalanced, the ball is experiencing acceleration, which will cause it to speed up going down.

## BFPM Practice and Exploration #2 (for Physics First)

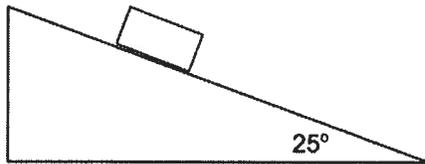
- Determine the tension in each cable:



- Determine the tension in each cable: (Hint--there is more than one way to define the system.)

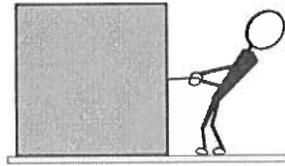


- A 200 N box slides down a  $25^\circ$  ramp at a constant speed. Which of the following could NOT possibly be the magnitude of the frictional force? Explain why.

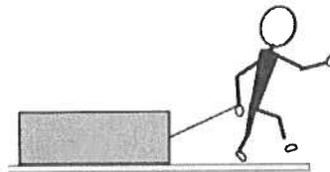


- 1 N
- 84 N
- 200 N
- 220 N

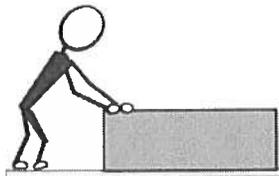
- A man pulls a 50 kg box at a constant speed across the floor using a rope held horizontal to the floor. He applies a 150 N force.



- On the left side of the picture, draw a FBD for the box in this situation.
- On the right side of the picture, draw a FBD for the man in this situation.
- What is the normal force exerted by the floor on the box? Show how you know this.
- What is the frictional force exerted by the floor on the box? Show how you know this.
- Imagine that the rope was pulled at an angle above the floor, as shown below, but everything else stayed the same.



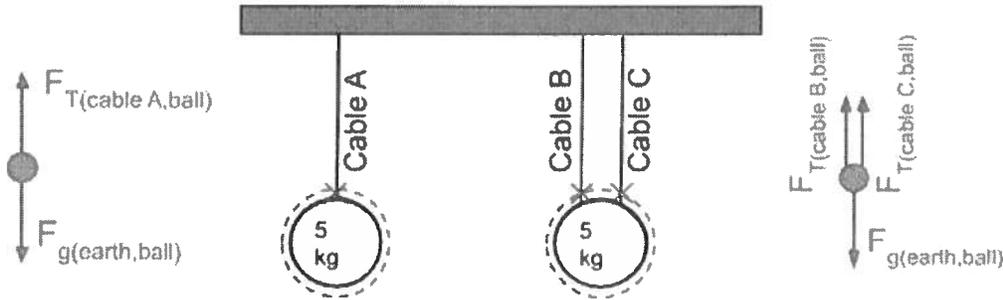
- On the left side of the picture, draw a FBD for the box in this situation.
- How would this change the normal force exerted by the floor on the box? Explain.
- Imagine that the box was pushed at a downward angle, as shown below.



- On the right side of the picture, draw a FBD for the box in this situation.
- How would this change the normal force by the floor on the box? Explain.

## BFPM Practice and Exploration #2 (for Physics First) Solutions

- Determine the tension in each cable:



A free body diagram that describes the object on the left is shown on the left above and on the right is a FBD that describes the object on the right. If we assume that both these objects are in

a constant state of motion, then, according to NL1,  $\sum F = 0$ . In both cases, then,

$\sum F_{up} = \sum F_{down}$ . So in the case on the left...  $F_T = F_g$ . Since

$$F_g = \left( \frac{5 \text{ kg}}{1 \text{ kg}} \right) \cdot \left( \frac{10 \text{ N}}{1 \text{ kg}} \right) = 50 \text{ N}$$

, this means that the tension in cable A is 50 N.

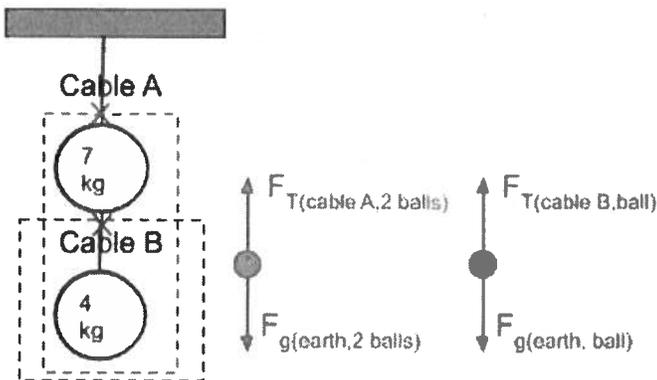
In the case on the right, according to NL1,  $F_{TB} + F_{TC} = F_g$ . There is no reason to believe that either of these two cables supports more of the weight than the other, since they seem to be

identically placed, so let's say  $F_{TB} = F_{TC} = F_T$ , which means that  $2F_T = F_g = 50 \text{ N}$ , so

$$2F_T = \frac{50 \text{ N}}{2} = 25 \text{ N}$$

, so the tension in each of cable B and cable C is 25 N.

- Determine the tension in each cable: (Hint--there is more than one way to define the system.)



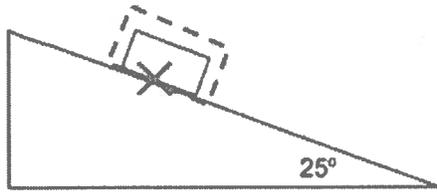
If we assume both balls are in a

constant state of motion, then NL1 implies that  $\sum F_{up} = \sum F_{down}$ . In the red system,

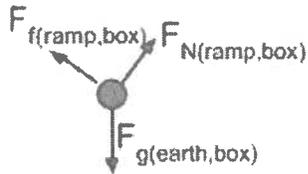
that means  $F_{TA} = F_g = 70 \text{ N} + 40 \text{ N} = 110 \text{ N}$ . In the blue system this means

$$F_{TB} = F_g = 40 \text{ N}$$

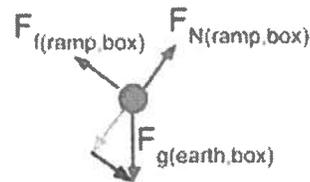
- A 200 N box slides down a 25° ramp at a constant speed. Which of the following could NOT possibly be the magnitude of the frictional force? Explain why.



Here's the basic free body diagram for the box.



But the force of gravity both holds the box onto the ramp and pulls the box down the ramp. We can think about  $F_g$  being replaced by two forces, then: one down the



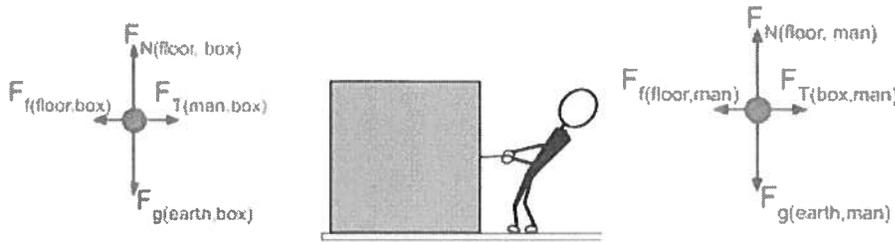
ramp (shown in blue) and one into the ramp (shown in green)

The size

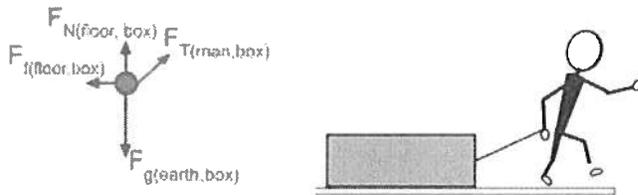
of the blue part of gravity's force could not possibly be more than the weight of the box ( $F_g$ ), no matter how steep the ramp is. Because the forces in the axis parallel to the ramp have to be balanced,  $F_f$  is the same as the blue part of  $F_g$ , so the only force that could not possibly be the value of  $F_f$  is the one that is greater than the  $F_g$ : 220 N.

- 1 N
- 84 N
- 200 N
- 220 N

- A man pulls a 50 kg box at a constant speed across the floor using a rope held horizontal to the floor. He applies a 150 N force.

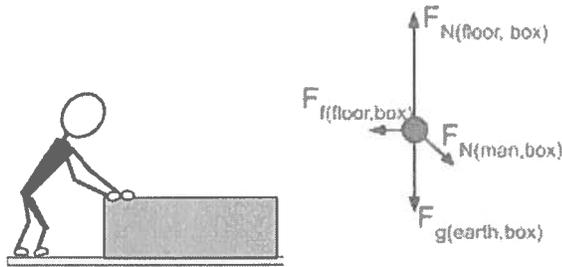


- On the left side of the picture, draw a FBD for the box in this situation.
- On the right side of the picture, draw a FBD for the man in this situation.
- What is the normal force exerted by the floor on the box? Show how you know this. Because the box is in a constant state of motion,  $\sum F = 0$ . In the vertical axis, this means that  $F_N = F_g$ . But  $F_g = \left(\frac{50 \text{ kg}}{1 \text{ kg}}\right) \cdot \left(\frac{10 \text{ N}}{1 \text{ kg}}\right) = 500 \text{ N}$ . So the normal force exerted by the floor is 50 N.
- What is the frictional force exerted by the floor on the box? Show how you know this. Because  $\sum F = 0$ , in the horizontal axis,  $F_f = F_T = 150 \text{ N}$
- Imagine that the rope was pulled at an angle above the floor, as shown below, but everything else stayed the same.



- On the left side of the picture, draw a FBD for the box in this situation.
- How would this change the normal force exerted by the floor on the box? Explain. If the man pulls at an upward angle, the  $F_T$  he exerts does two things. First, it pulls the object to the right, like in the previous picture. However, it also partly pulls upward. Since this upward force plus the  $F_N$  provided by the floor both add up to a total that is equal and opposite the  $F_g$ , the  $F_N$  provided by the floor must now be smaller than  $F_g$ .

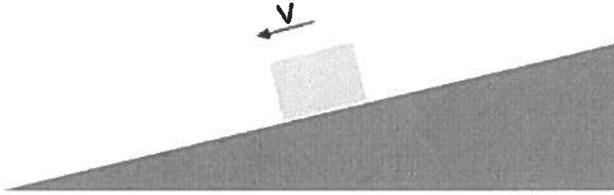
- Imagine that the box was pushed at a downward angle, as shown below.



- On the right side of the picture, draw a FBD for the box in this situation.
- How would this change the normal force by the floor on the box? Explain.  
If the man pushes at an downward angle, the  $F_N$  he exerts does two things. First, it pushes the object to the right. However, it also partly pushes downward. Since this downward force plus  $F_g$  both add up to a total that is equal and opposite the  $F_N$ , the  $F_N$  provided by the floor must now be larger than  $F_g$ .

### BFPM Practice and Exploration #3 (for Physics First)

- A 25 kg box is sliding down a ramp at a constant speed.



- Draw a free-body diagram for the box to the right of the sketch above.
- What is the gravitational force on the box?
- What is the range of possible values for the normal force by the ramp on the box? Explain how you know this.

$$\text{_____} < F_N < \text{_____}$$

- What is the range of possible values for the frictional force by the ramp on the box? Explain how you know this.

$$\text{_____} < F_f < \text{_____}$$

- A 3 kg lamp is supported by a solid horizontal rod that is connected to a wall by a hinge and a cable that is connected to the wall at an angle.



- Draw a free-body diagram for the lamp in the space to the right of the sketch.
- What is the gravitational force on the lamp?
- What is the least possible value for the tension force by the cable on the lamp? Explain how you know this.

$$\text{_____} < F_T$$

- Use a free-body diagram to help you explain why this arrangement would not work if the cable was attached to the wall at the same place as the rod.

- A kid pulls a 12 kg wagon across the ground at a constant speed by exerting a force of 50 N at an upward angle to the ground.



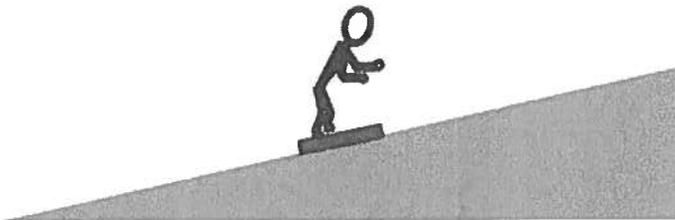
- Draw a free-body diagram for the wagon in the space to the right of the sketch.
- What is the range of possible values for the frictional force by the ground on the wagon? Explain how you know this.

$$\text{_____} < F_f < \text{_____}$$

- What is the range of possible values for the normal force by the ground on the wagon? Explain how you know this.

$$\text{_____} < F_N < \text{_____}$$

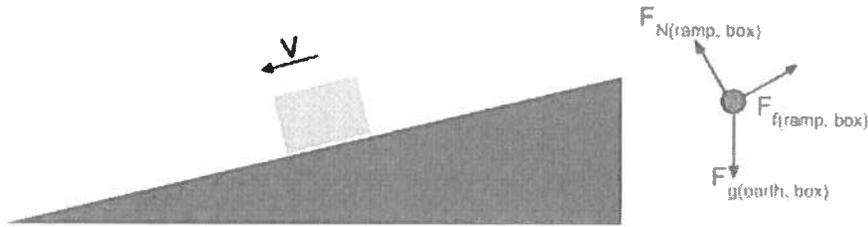
- A man stands on a scale on the pitched roof of his house. The scale reads 780 N. (It's a weird scale he stole from a laboratory.)



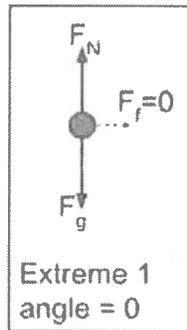
- Draw a free-body diagram for the man.
- What is the most this man's mass could possibly be, in kilograms? Explain how you know this.

### BFPM Practice and Exploration #3 (for Physics First) Solutions

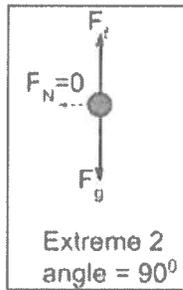
- A 25 kg box is sliding down a ramp at a constant speed.



- Draw a free-body diagram for the box to the right of the sketch above.
- What is the gravitational force on the box?  $\left(\frac{25 \text{ kg}}{1 \text{ kg}}\right) \cdot \left(\frac{10 \text{ N}}{1 \text{ kg}}\right) = 250 \text{ N}$
- What is the range of possible values for the normal force by the ramp on the



box? Explain how you know this. In the case of this extreme,



$F_N = F_g = 250 \text{ N}$  In the case of this extreme,

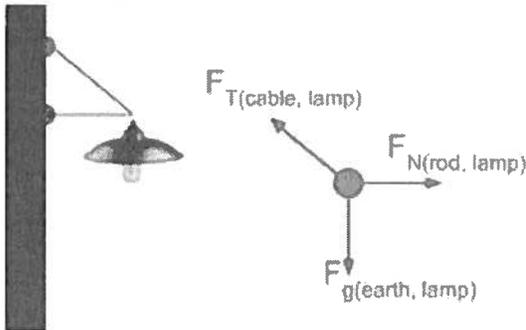
$$F_f = F_g = 250 \text{ N}$$

$$0 < F_N < 250 \text{ N}$$

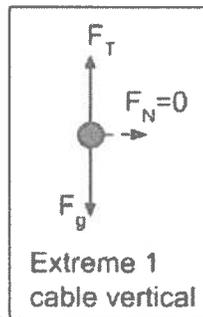
- What is the range of possible values for the frictional force by the ramp on the box? Explain how you know this.

$$0 < F_f < 250 \text{ N}$$

- A 3 kg lamp is supported by a solid horizontal rod that is connected to a wall by a hinge and a cable that is connected to the wall at an angle.



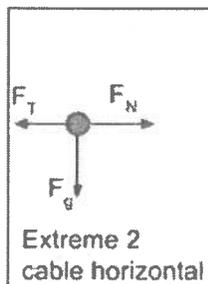
- Draw a free-body diagram for the lamp in the space to the right of the sketch.
- What is the gravitational force on the lamp?  $\left(\frac{3 \text{ kg}}{1 \text{ kg}}\right) \cdot \left(\frac{10 \text{ N}}{1 \text{ kg}}\right) = 30 \text{ N}$
- What is the least possible value for the tension force by the cable on the lamp?



Explain how you know this. In this extreme, the cable balances only the  $F_g$ , so this is the minimum tension in the cable:  $F_T = F_g = 30 \text{ N}$

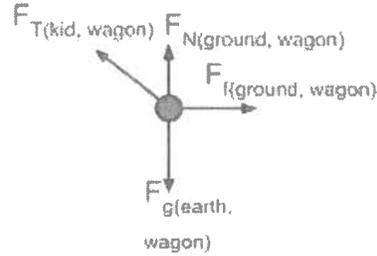
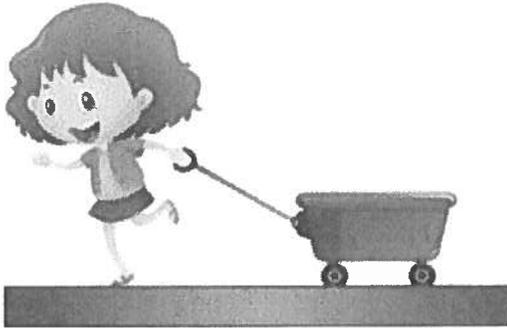
30 N <  $F_T$

- Use a free-body diagram to help you explain why this arrangement would not work if the cable was attached to the wall at the same place as the rod.

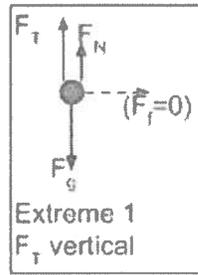


This extreme is impossible, because there is no upward vertical force to balance the  $F_g$ .

- A kid pulls a 12 kg wagon across the ground at a constant speed by exerting a force of 50 N at an upward angle to the ground.



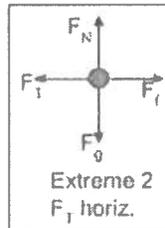
- Draw a free-body diagram for the wagon in the space to the right of the sketch.
- What is the range of possible values for the frictional force by the ground on the



wagon? Explain how you know this.

$$F_T + F_N = F_g, \text{ so } 50 \text{ N} + F_N = 120 \text{ N}, \text{ so}$$

In this extreme,



$$F_N = 120 \text{ N} - 50 \text{ N} = 70 \text{ N}$$

$$F_N = F_g = 120 \text{ N} \text{ and } F_f = F_T = 50 \text{ N}$$

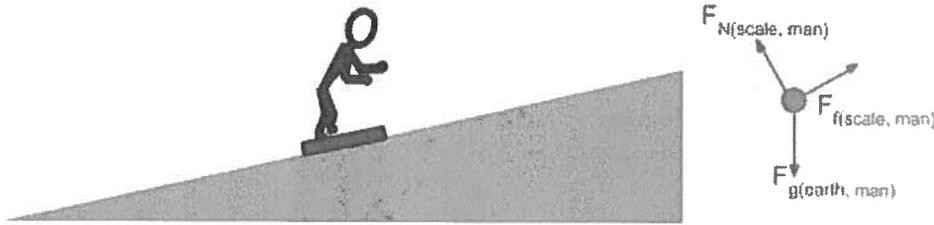
In this extreme,

$$\underline{0} < F_f < \underline{50 \text{ N}}$$

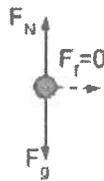
- What is the range of possible values for the normal force by the ground on the wagon? Explain how you know this.

$$\underline{70 \text{ N}} < F_N < \underline{120 \text{ N}}$$

- A man stands on a scale on the pitched roof of his house. The scale reads 780 N. (It's a weird scale he stole from a laboratory.)



- Draw a free-body diagram for the man.
- What is the most this man's mass could possibly be, in kilograms? Explain how you know this. The scale doesn't actually read the man's weight, it reads the normal force it exerts on the man. In every case where the angle of the roof is not zero, the normal force will be less than the gravitational force, because both the normal force and the frictional force combine to balance the gravitational force (which is the man's weight). So the extreme where the roof is flat gives the



upper limit on his weight:

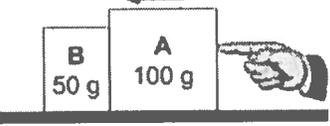
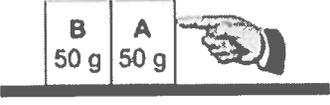
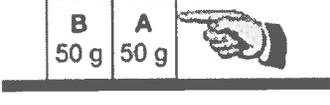
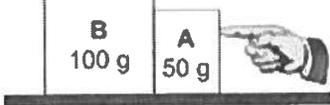
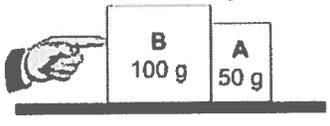
In this case,  $F_N = F_g = 780 \text{ N}$ , so his

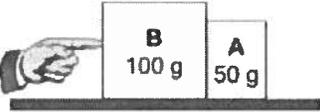
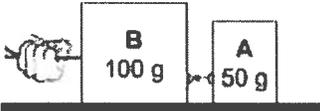
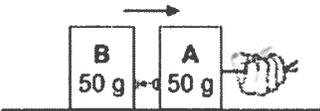
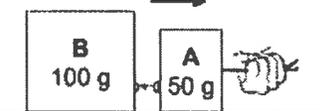
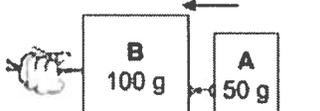
mass in that case will be:  $\left(\frac{780 \text{ N}}{10 \text{ N}}\right) \cdot \left(\frac{1 \text{ kg}}{10 \text{ N}}\right) = 78 \text{ kg}$  Therefore, whatever the man's actual mass, it is less than 78 kg.

### BFPM Practice and Exploration #4

In each row of the table below, there is a sketch of a pair of blocks interacting.

- In the box immediately to the left, draw the FBD for the block on the left in the sketch.
- In the box immediately to the right, draw the FBD for the block on the right in the sketch.
- In the far right box, list NL3 interacting pairs.
- In the far left, put a "<", "=", or ">" in the blank to compare the interacting forces compare.

$F_{A \text{ on } B}$ _____ $F_{B \text{ on } A}$		constant velocity 		
$F_{A \text{ on } B}$ _____ $F_{B \text{ on } A}$		constant velocity 		
$F_{A \text{ on } B}$ _____ $F_{B \text{ on } A}$		constant acceleration 		
$F_{A \text{ on } B}$ _____ $F_{B \text{ on } A}$		constant velocity 		
$F_{A \text{ on } B}$ _____ $F_{B \text{ on } A}$		constant velocity 		

$F_{A \text{ on } B} \text{ \_\_\_\_ } F_{B \text{ on } A}$		<p style="text-align: center;">constant acceleration</p> 		
$F_{A \text{ on } B} \text{ \_\_\_\_ } F_{B \text{ on } A}$		<p style="text-align: center;">constant velocity</p> 		
$F_{A \text{ on } B} \text{ \_\_\_\_ } F_{B \text{ on } A}$		<p style="text-align: center;">constant acceleration</p> 		
$F_{A \text{ on } B} \text{ \_\_\_\_ } F_{B \text{ on } A}$		<p style="text-align: center;">constant acceleration</p> 		
$F_{A \text{ on } B} \text{ \_\_\_\_ } F_{B \text{ on } A}$		<p style="text-align: center;">constant acceleration</p> 		

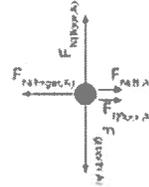
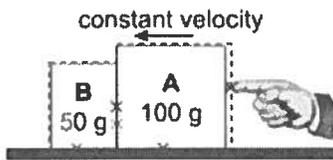
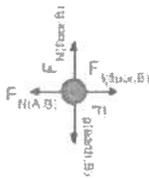
## BFPM Practice and Exploration #4 Solutions

**Note: I have only included the interacting NL3 pairs between A and B. There are actually more pairs that could be included in a more thorough list.**

Because of NL1, the net force = 0, so in both FBD's the forces should balance. For block A, this means

$$F_{N(\text{finger}, A)} = F_{N(B, A)} + F_{f(\text{floor}, A)}$$

$$F_{A \text{ on } B} = -F_{B \text{ on } A}$$

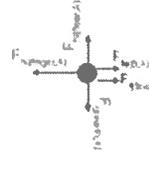
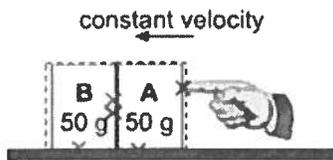
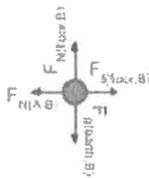


$$F_{N(A, B)}$$
  
and  
$$F_{N(B, A)}$$

Because of NL1, the net force = 0, so in both FBD's the forces should balance. For block A, this means

$$F_{N(\text{finger}, A)} = F_{N(B, A)} + F_{f(\text{floor}, A)}$$

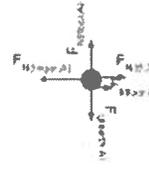
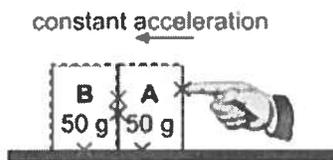
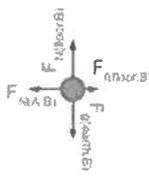
$$F_{A \text{ on } B} = -F_{B \text{ on } A}$$



$$F_{N(A, B)}$$
  
and  
$$F_{N(B, A)}$$

Because of NL1, net force  $\neq 0$ , so in both FBD's the forces should be unbalanced toward the right. For block A, this means  $F_{N(\text{finger}, A)} > F_{N(B, A)} + F_{f(\text{floor}, A)}$ . For block B, this means  $F_{N(A, B)} > F_{f(\text{floor}, B)}$ .

$$F_{A \text{ on } B} = -F_{B \text{ on } A}$$

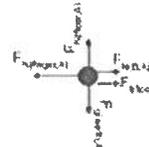
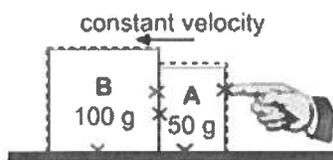
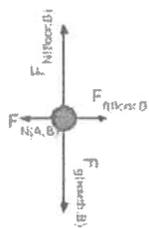


$$F_{N(A, B)}$$
  
and  
$$F_{N(B, A)}$$

Because of NL1, the net force = 0, so in both FBD's the forces should balance. For block A, this means

$$F_{N(\text{finger}, A)} = F_{N(B, A)} + F_{f(\text{floor}, A)}$$

$$F_{A \text{ on } B} = -F_{B \text{ on } A}$$



$$F_{N(A, B)}$$
  
and  
$$F_{N(B, A)}$$

Because of NL1, the net force = 0, so in both FBD's the forces should balance.

$F_{A \text{ on } B} = -F_{B \text{ on } A}$		<p>constant velocity</p>		$F_{N(A, B)}$ and $F_{N(B, A)}$
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Because of NL1, net force  $\neq 0$ , so in both FBD's the forces should be unbalanced toward the right. For block B, this means  $F_{N(\text{finger}, B)} > F_{N(A, B)} + F_{f(\text{floor}, B)}$ . For block A this means  $F_{N(B, A)} > F_{f(\text{floor}, A)}$ .

$F_{A \text{ on } B} = -F_{B \text{ on } A}$		<p>constant acceleration</p>		$F_{N(A, B)}$ and $F_{N(B, A)}$
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Because of NL1, the net force = 0, so in both FBD's the forces should balance. For block B, this means  $F_{T(\text{hand}, B)} = F_{T(A, B)} + F_{f(\text{floor}, B)}$ .

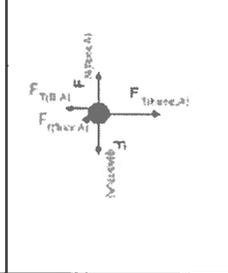
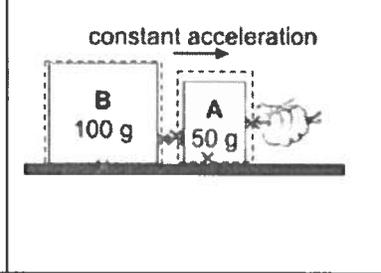
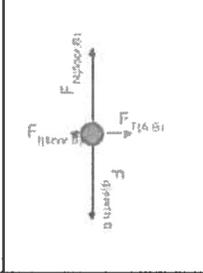
$F_{A \text{ on } B} = -F_{B \text{ on } A}$		<p>constant velocity</p>		$F_{T(A, B)}$ and $F_{T(B, A)}$
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Because of NL1, net force  $\neq 0$ , so in both FBD's the forces should be unbalanced toward the right. For block A, this means  $F_{T(\text{hand}, A)} > F_{T(B, A)} + F_{f(\text{floor}, A)}$ . For block B, this means  $F_{T(A, B)} > F_{f(\text{floor}, B)}$ .

$F_{A \text{ on } B} = -F_{B \text{ on } A}$		<p>constant acceleration</p>		$F_{T(A, B)}$ and $F_{T(B, A)}$
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Because of NL1, net force  $\neq 0$ , so in both FBD's the forces should be unbalanced toward the right. For block A, this means  $F_{T(hand,A)} > F_{T(B,A)} + F_{f(floor,A)}$ . For block B, this means  $F_{T(A,B)} > F_{f(floor,B)}$ .

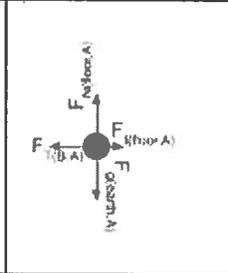
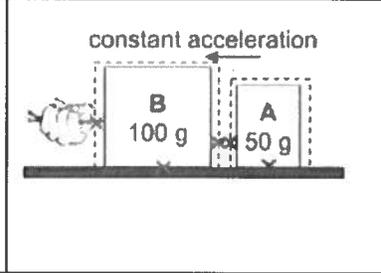
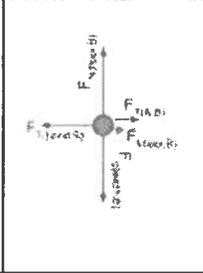
$F_{A \text{ on } B} = F_{B \text{ on } A}$



$F_{T(A,B)}$   
and  
 $F_{T(B,A)}$

Because of NL1, net force  $\neq 0$ , so in both FBD's the forces should be unbalanced toward the right. For block B, this means  $F_{T(hand,B)} > F_{T(A,B)} + F_{f(floor,B)}$ . For block A, this means  $F_{T(B,A)} > F_{f(floor,A)}$ .

$F_{A \text{ on } B} = F_{B \text{ on } A}$



$F_{T(A,B)}$   
and  
 $F_{T(B,A)}$